New Higher Homework 9

Section 1: Vectors

1.   A, B and C have coordinates (2,-3,5), (8,3,2) and (12,7,0), respectively.
      Show that A, B and C are collinear and write down the ratio in which B divides AC.

2.   A, B and C have coordinates (-3,6,4), (0,11,2) and (4,3,5), respectively.
      Find the size of angle BAC.

3.   P is the point (-3,2,5) and Q is the point (2,-8,20). X divides PQ in the ratio $3:2$.
      Find the coordinates of X.

4. $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. Show that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Section 2: Recurrence Relations

1.   A sequence is defined by the equation $u_{n+1} = 0.9u_n + 2$, with $u_1 = 3$.
      (a)   Calculate u_2.
      (b)   What is the least value of n such that $u_n > 10$?
      (c)   Explain why this sequence has a limit and calculate this limit algebraically.

2.   A gardener feeds her trees weekly with "Bioforce, the wonder plant food". It is known that in a week the amount of plant food in the trees falls by about 25%.

      (a)   The trees contain no Bioforce initially and the gardener applies 1g of Bioforce to each tree every Saturday. Bioforce is only effective when there is continuously more than 2g of it in the tree. Calculate how many weekly feeds will be necessary before the Bioforce becomes effective.

      (b)   (i)   Write down a recurrence relation for the amount of plant food in the tree immediately after feeding.
             (ii)   If the level of Bioforce in the tree exceeds 5g, it will cause leaf burn.
                    Is it safe to continue feeding the trees indefinitely?

3.   A sequence is defined by the recurrence relation $u_{n+1} = au_n + b$ $(n > 0)$.
      Given $u_1 = 7$, $u_2 = 19$ and $u_3 = 43$ find the values of a and b.