New Higher Homework 6

Section 1: Polynomials

1. $f(x) = x^3 + 2x^2 - x + 4,\quad g(x) = 3x^4 + 3x^2 + 5x - 17,\quad h(x) = x^4 - 6x + 7,\quad k(x) = 9x^3 - 7x^2 + 9.$

   Evaluate, using "synthetic division": (a) $f(3)$ (b) $g(-3)$ (c) $h(-2)$ (d) $k(2)$

2. Show that the following polynomials have the given factors, and hence factorise them fully.

   (a) $x^3 - 2x^2 - x + 2$; factor $(x-2)$ (b) $2x^3 - 11x^2 + 17x - 6$; factor $(x-3)$

4. Factorise fully: (a) $2x^3 + 5x^2 - 4x - 3$ (b) $2x^3 + 7x^2 + 2x - 3$ (c) $x^3 - 1$

5. Find p, given that $(x+3)$ is a factor of $x^3 - x^2 + px + 15$.

6. $f(x) = x^3 - 2x^2 - 5x + 6,\ g(x) = x - 1.$

   (a) Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$.
   (b) Factorise fully $f(g(x))$.
   (c) $h(x) = \dfrac{1}{f(g(x))}$. For what values of x is h not defined?

Section 2: Calculus

1. Differentiate: (a) $\sqrt{x}\left(1-\sqrt{x}\right)$ (b) $\dfrac{1+x^2}{\sqrt{x}}$ (c) $\dfrac{4}{3x^2}$ (d) $\left(x-\dfrac{1}{x}\right)^2$

2. (a) Find the gradient of the tangent to the curve $y = x^2 + \dfrac{24}{x}$ at the point P(2,16).
   (b) The tangent at P meets the x-axis at A and the y-axis at B. Calculate the area of ΔAOB.

3. The loss of heat from a hot water tank is proportional to the total surface area of the tank. It is therefore important to design a tank with the minimum possible surface area.

   (a) For a closed cylindrical tank of volume $50\,000\text{cm}^3$, show that the total surface area is given by $\mathrm{A} = 2\pi r^2 + \dfrac{100\,000}{r}$ (cm^2).
   (b) Use differentiation to find the value of r corresponding to a stationary value of A and show that this value is a minimum.
   (c) Find the minimum value of A and determine whether this is greater than or less than the total surface area of a cubic tank with the same volume $50000\,\text{cm}^3$