

New Higher Homework 4

Section 1: Differentiation

1. Differentiate each of the following w.r.t. x :

(a) $y = 2x^3 - 4x^2$ (b) $y = x^2 + \frac{1}{x}$ (c) $y = x(x-2)^2$

(d) $y = \sqrt{x}$ (e) $y = \frac{1}{\sqrt{x}}$ (f) $y = \frac{1}{2x^3}$

2. Find the equation of the tangent to:

- (a) $y = 4x^3 + 2x$ at the point where $x = 1$.
(b) $y = x^3 - 4x$ at the point where $x = -1$.
(c) $y = (x-2)^2$ at the point where $x = 1$.

3. In each of the following, find $f'(x)$:

(a) $f(x) = \frac{3}{4x^2}$ (b) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ (c) $f(x) = \frac{x^2 + 1}{x}$

(d) $f(x) = \sqrt{x}(x+1)$ (e) $f(x) = \frac{1+x}{\sqrt{x}}$ (f) $f(x) = \frac{x^2 + 2x}{x}$

4. (a) $f(x) = x^2 + 3x$. Calculate the rate of change of $f(x)$ when $x = 3$.

(b) $y = \frac{x^4 - x}{x^3}$. Calculate the rate of change of y when $x = 1$.

Section 2: Functions

1. Find the composite function $g(f(x))$ for each of the following, expressing your answer in its simplest form.

(a) $f(x) = 2x + 1$ (b) $f(x) = 2x + 1$
 $g(x) = x^2 + 1$ $g(x) = \frac{1}{x^2}$ (c) $f(x) = x^2 + 2x + 1$
 $g(x) = 2x^2 + 4$

2. Find the inverse function $f^{-1}(x)$ for each of the following.

(a) $f(x) = 3x - 5$ (b) $f(x) = x^2 + 2$ (c) $f(x) = \frac{x+4}{7}$