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## 200 Higher Maths

## Exam Type Questions

## FORMULAE LIST

## Circle:

The equation $x^{2}+y^{2}+2 g x+2 f y+c=0$ represents a circle centre $(-g,-f)$ and radius $\sqrt{g^{2}+f^{2}-c}$.
The equation $(x-a)^{2}+(y-b)^{2}=r^{2}$ represents a circle centre $(a, b)$ and radius $r$.

## Scalar Product:

$\mathbf{a} . \mathbf{b}=|\mathbf{a}||\mathbf{b}| \cos \theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$ or

$$
\text { a.b }=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3} \text { where } \mathbf{a}=\left(\begin{array}{l}
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right) \text { and } \mathbf{b}=\left(\begin{array}{l}
b_{1} \\
b_{2} \\
b_{3}
\end{array}\right) .
$$

Trigonometric formulae:

$$
\begin{aligned}
\sin (A \pm B) & =\sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) & =\cos A \cos B \mp \sin A \sin B \\
\sin 2 A & =2 \sin A \cos A \\
\cos 2 A & =\cos ^{2} A-\sin ^{2} A \\
& =2 \cos ^{2} A-1 \\
& =1-2 \sin ^{2} A
\end{aligned}
$$

Table of standard derivatives:

| $f(x)$ | $f^{\prime}(x)$ |
| :---: | :---: |
| $\sin a x$ | $a \cos a x$ |
| $\cos a x$ | $-a \sin a x$ |

Table of standard integrals:

| $f(x)$ | $\int f(x) d x$ |
| :--- | :---: |
| $\sin a x$ | $-\frac{1}{a} \cos a x+c$ |
| $\cos a x$ | $\frac{1}{a} \sin a x+c$ |

Clear, easy to follow, step-by-step worked solutions to all 200 questions available in the Higher Maths Online Study Pack at www.highermathematics.co.uk

| 1 | Show that $(x-1)$ is a factor of $f(x)=2 x^{3}+x^{2}-8 x+5$. Hence fully factorise $f(x)$ fully. |  |
| :---: | :---: | :---: |
| 2 | Express $x^{2}+8 x+3$ in the form $(x+p)^{2}+q$ and state the coordinates of the turning point of the parabola. |  |
| 3 | Evaluate: $\log _{5} 2+\log _{5} 50-\log _{5} 4$ |  |
| 4 | What is the solution of the equation $2 \sin x-\sqrt{3}=0$ where $\frac{\pi}{2} \leq x \leq \pi$ ? |  |
| 5 | Given that $0 \leq a \leq \frac{\pi}{2}$ and $\sin a=\frac{3}{5}$, find an expression for $\sin (x+a)$. |  |
| 6 | $\text { If }=4 x^{3}+5 x^{2}-3 x+2, \text { find } \frac{d y}{d x}$ |  |
| 7 | Find the coordinates of the turning points of the curve with equation $y=x^{3}-3 x^{2}-9 x+12$ <br> and determine their nature. |  |
| 8 | Find $\int\left(2 x^{-4}+\cos 5 x\right) d x$. |  |
| 9 | $\frac{d y}{d x}=8 x-3 . \text { If } y=7 \text { when } x=2$ <br> find an equation for $y$. |  |
| 10 | The expression $\sqrt{3} \sin x^{\circ}-\cos x^{\circ}$ can be written in the form $\operatorname{ksin}(x-a)^{\circ}$, where $k>0$ and $0 \leq a<360$. Calculate the values of $k$ and $a$. |  |

11
A function $f$ is given by $f(x)=\sqrt{9-x^{2}}$.
What is a suitable domain of $f$ ?

12
The diagram shows the graph with equation of the form $y=a \operatorname{cosb} x$ for $0 \leq x \leq 2 \pi$. What is the equation of this graph?
$E(-2,-1,4), P(1,5,7)$ and $F(7,17,13)$ are three collinear points. P lies between E and $F$. What is the ratio in which $P$ divides EF?

14
Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}|=3$,
$|\boldsymbol{q}|=4$ and $\boldsymbol{p} \cdot \boldsymbol{q}=10$.
Find the value of $\boldsymbol{q} \cdot(\boldsymbol{p}+\boldsymbol{q})$.
15
Write down the exact values of $\sin 60^{\circ}$ and
$\tan \frac{\pi}{6}$.

16 The diagram shows a line $L$; the angle between L and the positive direction of the $x$-axis is $135^{\circ}$, as shown.
What is the gradient of the line L?
17 The vertices of triangle $A B C$ are
$A(7,9), B(-3,-1)$ and $C(5,-5)$
as shown in the diagram.
Find the equation of the median from C .
$A(7,9), B(-3,-1)$ and $C(5,-5)$
as shown in the diagram.
Find the equation of the median from C .
$A(7,9), B(-3,-1)$ and $C(5,-5)$
as shown in the diagram.
Find the equation of the median from C .
18
The $x$-axis is a tangent to a circle with centre $(-7,6)$ as shown in the diagram.
What is the equation of the circle?


19 A sequence is defined by the recurrence
relation

$$
u_{n+1}=0.3 u_{n}+6 \text { with } u_{10}=10
$$

What is the value of $u_{12}$ ?
20 The diagram shows graphs with equations $y=14-x^{2}$ and $y=2 x^{2}+2$. Calculate the shaded area.


## 21 <br> Show that $(x-1)$ is a factor of $x^{3}-3 x+2$.

Hence or otherwise factorise $x^{3}-3 x+2$ fully.
22
$2 x^{2}+4 x+7$ is expressed in the form
$2(x+p)^{2}+q$. What is the value of $q$.
23
If $\log _{4} 12-\log _{4} x=\log _{4} 6$, what is the value of $x$ ?

## 24

Solve $2 \cos x=\sqrt{3}$ for $x$, where $0 \leq x<2 \pi$.

25
If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2 x$.

26
Given that $f(x)=\left(4-3 x^{2}\right)^{\frac{-1}{2}}$ on a suitable domain, find $f^{\prime}(x)$.

27
Find the coordinates of the stationary points on the curve $f(x)=x^{3}-3 x+2$ and determine their nature.

28
Find $\int\left(4 x^{\frac{1}{2}}+x^{-3}\right) d x$, where $x>0$.

29
The graph of $y=f(x)$ passes through the point
$\left(\frac{\pi}{9}, 1\right)$. If $f^{\prime}(x)=\sin (3 x)$ express $y$ in terms of $x$.

30
Write $\sin x-\cos x$ in the form $k \sin (x-a)$ stating the values of $k$ and $a$ where $k>0$ and $0 \leq a \leq 2 \pi$.

31 Functions $f$ and $g$ are given by $f(x)=3 x+1$
and $g(x)=x^{2}-2$.
Find $f(g(x))$ and $g(f(x))$.
32 The diagram shows the graph of $y=f(x)$ where $f$ is a logarithmic function. What are the values of a and b for

$(x)=\log _{a}(x-b)$ ?
33
The vectors $\boldsymbol{u}=\left(\begin{array}{c}k \\ -1 \\ 1\end{array}\right)$ and $\boldsymbol{v}=\left(\begin{array}{l}0 \\ 4 \\ k\end{array}\right)$ are
perpendicular. What is the value of $k$ ?
$34 D, E$ and $F$ have coordinates $(10,-8,-15)$, $(1,-2,-3)$ and $(-2,0,1)$ respectively.
Show that D, E and F are collinear and find the ratio in which E divides DF.

35
Prove that $\frac{\cos ^{3} x}{1-\sin ^{2} x}=\cos x$.

36 The line $L$ passes through the point $(-2,-1)$ and is parallel to the line with equation
$5 x+3 y-6=0$.
What is the equation of L ?
37
Triangle PQR has vertices at $P(-3,-2), Q(-1,4)$
and $R(3,6)$. PS is a median.
What is the gradient of PS?
38 The diagram shows a circle, centre $(2,5)$ and a tangent drawn at the point $(7,9)$. What is the equation of this tangent?


39 A sequence is generated by the recurrence relation $u_{n+1}=0.4 u_{n}-240$.
What is the limit of this sequence as $\rightarrow \infty$ ?

40 Calculate the shaded area enclosed by the curve $y=x^{3}(3-x)$ and the $x$-axis between $x=0$ and $x=3$.


41 The graph has an equation of the form $y=k(x-a)(x-b)$. What is the equation of the graph?


42
For what values of $x$ is $6+x-x^{2}<0$ ?

## 43

Express $\log _{a} 25+\log _{a} 4-\log _{a} 20$ as the logarithm of a single number.

## 44

Solve $\cos 2 x-3 \cos x+2=0$ for $0 \leq x<360$.

45
The diagram shows two rightangled triangles with sides and angles given. What is the value of $\sin (p+q)$ ?


46
What is the derivative of $\left(x^{3}+4\right)^{2}$ ?

47
The point $P(5,12)$ lies on the curve with equation $y=x^{2}-4 x+7$. Find the
equation of the tangent to the curve.

48
Find $\int 4 \sin (2 x+3) d x$.

49
Find $\int_{-2}^{2}(x+1)^{2} d x$

50
Write $2 \sin x^{\circ}+3 \cos x^{\circ}$ in the form
$k \sin (x-a)$, for $k>0$ and $0 \leq a \leq 360$.

51 Functions $f$ and $g$ are defined on a suitable
domain by $f(x)=\cos x$ and $g(x)=x+\frac{\pi}{6}$. What
is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$ ?
52
The diagram shows the graph of $y=f(x)$.
Sketch $y=f(x+2)-1$


53
Given that $\boldsymbol{u}=\left(\begin{array}{l}2 \\ 0 \\ 1\end{array}\right)$ and $\boldsymbol{v}=\left(\begin{array}{c}-1 \\ 2 \\ 4\end{array}\right)$, find
$3 \boldsymbol{u}-2 \boldsymbol{v}$ in component form.
54
The vectors $x \boldsymbol{i}+5 \boldsymbol{j}+7 \boldsymbol{k}$ and $-3 \boldsymbol{i}+2 \boldsymbol{j}-\boldsymbol{k}$ are perpendicular. What is the value of $x$ ?

55
Prove that $2 \cos ^{2} A+3 \sin ^{2} A-2=\sin ^{2} A$.

56 A line makes an angle of $30^{\circ}$ with the positive direction of the $x$-axis as shown. What is the gradient of the line?
57
Find the equation of the perpendicular bisector
of the line joining $P(3,-3)$ to $Q(-1,9)$.

58
Write down the centre and calculate the radius of the circle with equation

$$
x^{2}+y^{2}+8 x+4 y-38=0
$$

59
A sequence is defined by the recurrence relation

$$
u_{n+1}=2 u_{n}+3 \text { and } u_{0}=1 .
$$

What is the value of $u_{3}$ ?
60 Calculate the shaded area enclosed by the line $y=2 x-3$ and the curve $y=x^{2}-5 x-3$.


61
Show that $x=1$ is a root of
$x^{3}+8 x^{2}+11 x-20=0$.
Hence factorise $x^{3}+8 x^{2}+11 x-20$ fully.

62
The roots of the equation $k x^{2}-3 x+2=0$ are equal.
Calculate the value of $k$.
63
Evaluate $\log _{2} \frac{1}{16}$.

64
Solve the equation $3 \cos 2 x+\cos x=-1$
in the interval $0 \leq x \leq 360$.

65 The diagram shows a right-angled triangle with sides and angles marked. What is the value of $\cos 2 a$ ?

${ }^{66} A=2 \pi r^{2}+6 \pi r$.
What is the rate of change of $A$ with repect to $r$ when $r=2$ ?

67
Find the equation of the tangent to the curve
$y=x^{3}-3 x^{2}+2 x$ at the point where $\mathrm{x}=1$.

68
Find $\int \frac{1}{3 x^{4}} d x$, where $x \neq 0$.

69
Evaluate $\int_{0}^{\frac{\pi}{2}} \sin 2 x+\cos 2 x d x$

70
Write $3 \cos x^{\circ}+4 \sin x^{\circ}$ in the form $k \cos (x+a)$
for $k>0$ and $0 \leq x \leq 360$

71 Functions $f$ and $g$ are defined on the set of real numbers by $f(x)=x^{2}+3$ and $g(x)=x+4$. Find expressions for $f(g(x))$ and $g(f(x))$.

72 The diagram shows part of the ${ }^{y}$ graph of $y=\log _{3}(x-4)$. The point $(q, 2)$ lies on the graph. What is the value of $q$ ?
73
Given that the ratio $S(-4,5,1), T(-16,-4,16)$ and $U(-24,-10,26)$ are collinear, calculate the ratio in which $T$ divides $S U$.

74 An equilateral triangle of side 3 units is shown. The vectors $\mathbf{p}$ and $\mathbf{q}$ are as represented in the diagram. What is the value of p.q?


75
Convert $135^{\circ}$ into radians and convert $\frac{2 \pi}{3}$ into degrees.

## 76

Calculate the distance between the points ( $4,-1$ ) and $(7,3)$.

77
A triangle has vertices $P(1,8), Q(-12,-2)$ and $R(8,-6)$. Calculate the median PS.

78
The line with equation $y=2 x$ intersects the circle with equation $x^{2}+y^{2}=5$ at the points J and K. What are the $x$-coordinates of J and K ?

A sequence is generated by the recurrence
relation $u_{n+1}=0.7 u_{n}+10$.
What is the limit of this sequence as $n \rightarrow \infty$ ?

80
Calculate the shaded area shown in the diagram.


## 81

The diagram shows the graph with equation
$y=k(x-1)^{2}(x+t)$.
What are the values of $k$ and $t$ ?


82
What is the solution of $x^{2}+4 x>0$, where $x$ is a real number?

83
Find $x$ if $\log _{x} 6-2 \log _{x} 4=1$.

84
Solve the equation $\sin 2 x-\cos x=0$ in the interval $0 \leq x \leq 180$.

## 85

If $a$ and $b$ are acute angles such that sina $=\frac{4}{5}$ and $\sin b=\frac{5}{13}$, find the value of $\sin (a+b)$.

86
If $f(x)=\frac{1}{\sqrt[5]{x}}, x \neq 0$, what is $f^{\prime}(x)$ ?

87 Find the equation of the tangent to the curve with equation $y=x^{3}+2 x^{2}-3 x+2$ at the point where $x=1$.

88
Find $\int(2 x-1)^{\frac{1}{2}} d x$ where $x>\frac{1}{2}$.

89
Find $\int_{0}^{1} \frac{d x}{(3 x+1)^{\frac{1}{2}}}$

89
Express $3 \cos x^{\circ}+5 \sin x^{\circ}$ in the form $k \cos \left(x-a^{\circ}\right)$ where $k>0$ and $0 \leq a \leq 90$.

91
The functions $f$ and $g$ are defined by $f(x)=x^{2}+1$ and $g(x)=3 x-4$, on the set of real numbers. Find $f(g(x))$ and $g(f(x))$.

92 The diagram shows a sketch of a trig function whose equation is of the form $y=\operatorname{asin}(b x)+c$.
Determine the values of $a, b$ and $c$.


## 93

Show that the points $A(-7,-8,1), T(3,2,5)$
and $B(18,17,11)$ are collinear.
Find the ratio in which $T$ divides $A B$.
$94 P, Q$ and $R$ have coordinates $(1,3,-1),(2,0,1)$ and $(-3,1,2)$ respectively.
Express the vectors $\overrightarrow{Q P}$ and $\overrightarrow{Q R}$ in component form.
Hence or otherwise find the size of angle PQR.
95
Find the exact value TAN $\frac{7 \pi}{4}$

96
Find the equation of the line which passes
through the point $(-1,3)$ and is perpendicular to the line with equation $4 x+y-1=0$.

## 97

A triangle has vertices $\mathrm{A}(-3,1), \mathrm{B}(4,3)$ and $\mathrm{C}(6,-5)$.
Find the equation of the altitude BP.

## 98

A circle $C_{1}$ has equation $x^{2}+y^{2}+2 x+4 y-27=0$.
Write down the centre and calculate the radius of $C_{1}$.

## 99

A sequence is generated by the recurrence
relation $u_{n+1}=\frac{1}{4} u_{n}+7$, with $u_{0}=-2$. What is
the limit of this sequence as $n \rightarrow \infty$ ?
100
Calculate the shaded area shown in the diagram.


101 A function $f$ is defined on the set of real numbers by $f(x)=x^{3}-x^{2}+x+3$. What is the remainder when $f(x)$ is divided by $(x-1)$ ?

## 102

If $x^{2}-8 x+7$ is written in the form $(x-p)^{2}+q$, what is the value of $q$ ?

## 103

Given that $\log _{10} x=3 \log _{10} y+\log _{10} 2$, express $x$ in terms of $y$.

## 104

Solve the equation $2 \cos 3 x=1$, for $0 \leq x \leq 360$

105 The diagram shows a rightangled triangle with sides and angles marked. Find the value of $\sin 2 x$.


## 106

If $s(t)=t^{2}-5 t+8$, what is the rate of change of $s$ with respect to $t$ when $t=3$ ?

107 The diagram shows part of the graph of the curve $y=2 x^{3}-7 x^{2}+4 x+4$. Find the $x$-coordinate of the maximum turning point.


108
Find $\int x(3 x+2) d x$.

109
Find $f(x)$ given that $f^{\prime}(x)=2-\frac{1}{x^{2}}$ and $f(1)=8$.

## 110

Write $1.5 \cos x^{\circ}+2 \sin x^{\circ}$ in the form
$k \cos (x+a)^{\circ}$, where $0 \leq a \leq 180$.

111
A function $f$ is defined on a suitable domain by
$f(x)=\frac{x+2}{x^{2}-7 x+12}$.
What value(s) of $x$ cannot be in this domain?

## 112

The graph of $y=f(x)$ is shown. Sketch the graphs of $y=-f(x)$ and $y=-f(x)+3$.


113
The point Q divides the line joining $P(-1,-1,0)$ to $R(5,2,-3)$ in the ratio $2: 1$. Find the coordinates of $Q$.

114
If $\boldsymbol{u}=\left(\begin{array}{c}-3 \\ 1 \\ 2 t\end{array}\right)$ and $\boldsymbol{v}=\left(\begin{array}{c}1 \\ t \\ -1\end{array}\right)$ are perpendicular,
what is the value of $t$ ?
115
Prove the identity:
$2 \cos ^{2} x-1=1-2 \sin ^{2} x$

## 116

A line makes an angle of $45^{\circ}$ with the positive direction of the $x$-axis. What is its gradient?

117
Triangle ABC has vertices $A(-1,6), B(-3,-2)$ and $C(5,2)$. Find the equation of the line $q$, the perpendicular bisector of $B C$.

## 118

The point $\mathrm{P}(2,3)$ lies on the circle $(x+1)^{2}+(y-1)^{2}=13$.
Find the equation of the tangent at $P$.
119
A sequence is defined by the recurrence relation
$u_{n+1}=\frac{1}{3} u_{n}+1$, with $u_{2}=15$.
What is the value of $u_{4}$ ?
120
Calculate the area enclosed between the curves
$y=x^{2}-x+3$ and
$y=3+2 x-x^{2}$.


## 121

The diagram shows the graph of a cubic.
What is the equation of this cubic?

## 122

If $f(x)=(x-3)(x+5)$, for what values of
$x$ is the graph of $y=f(x)$ above the $x$-axis?

123
Simplify $5 \log _{8} 2+\log _{8} 4-\log _{8} 16$
124
Solve $4 \sin ^{2} x=3$ for $0 \leq x \leq 360$.

125
If $\cos A=\frac{5}{13}$ and $\sin B=\frac{4}{5}$, show that $\sin (A+B)=\frac{56}{65}$.

126
Given that $f(x)=4 \sin 3 x$, find $f^{\prime}(0)$.

## 127

A curve has equation $y=x-\frac{16}{\sqrt{x}}, x>0$.
Find the equation of the tangent at the point where $x=4$.

## 128

Find $\int(1-6 x)^{-\frac{1}{2}} d x$ where $x<\frac{1}{6}$.
$129 \frac{d y}{d x}=6 x^{2}-4 x+3$.
If $y=5$ when $x=1$, find an equation for $y$.

130
Express $8 \cos x^{\circ}-6 \sin x^{\circ}$ in the form $k \cos (x+a)^{\circ}$ where $k>0$ and $0<a<360$.

131
${ }^{31}(x)=3-x$ and $g(x)=\frac{3}{x}, x \neq 0$.
Find $p(x)=f(g(x))$.
If $q(x)=\frac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form.
132
The diagram shows $y=f(x)$. Sketch the graphs of $y=-2 f(x)$ and $y=f(x-3)$.


## 133

Show that the points $P(3,2,6), Q(5,-2,10)$ and $R(9,-10,18)$ are collinear.

## 134

Find the magnitude between the origin and the point 'a' $(3,4,0)$

## 135

Prove the identity:

$$
\cos A \tan A=\sin A
$$

## 136

Find the equation of the straight line through $(1,-7)$ perpendicular to the line $y-2 x=30$.

## 137

Find the equation of the median from C for a triangle with vertices $\mathrm{A}(1,-7), \mathrm{B}(-4,7)$ and $\mathrm{C}(-1,3)$.

## 138

Find the equation of the tangent to the circle
$x^{2}+y^{2}-10 y-43=0$ at the point $(2,-3)$.

## 139

A sequence is generated by the recurrence relation
$u_{n+1}=0.4 u_{n}-30$.
What is the limit of the sequence as $\rightarrow \infty$ ?
140
Calculate the shaded area shown in the diagram.


## 141

Show that $(x-4)$ is a factor of $x^{3}-5 x^{2}+2 x+8$.
Hence, fully factorise and solve $x^{3}-5 x^{2}+2 x+8$.

## 142

Solve $6-x-x^{2}<0$

## 143

Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A=A_{0} e^{k t}$ where $A_{0}$ is the area covered by the fire when it was first detected and $A$ is the area covered by the fire $t$ hours later. If it takes 1.5 hours for the area of the forest fire to double, find the value of the constant $k$.

## 144

Solve $2 \sin (2 x-60)^{\circ}=1$ for $0 \leq x \leq 360$.

## 145

Using $75^{\circ}=45^{\circ}+30^{\circ}$,

$$
\text { show that } \sin 75^{\circ}=\frac{\sqrt{6}+\sqrt{2}}{4} \text {. }
$$

146

$$
\text { If } y=3 x^{-2}+2 x^{\frac{3}{2}}, x>0 \text {, determine } \frac{d y}{d x}
$$

## 147

The parabola with equation $y=x^{2}-14 x+53$
has a tangent at the point $\mathrm{P}(8,5)$.
Find the equation of this tangent.

## 148

Find $\int \frac{\left(x^{2}-2\right)\left(x^{2}+2\right)}{x^{2}} d x, x \neq 0$

149
The curve $y=f(x)$ is such that $\frac{d y}{d x}=4 x-6 x^{2}$.
The curve passes through the point $(-1,9)$.
Express $y$ in terms of $x$.
150
Express $3 \cos x^{\circ}+4 \sin x^{\circ}$ in the form $k \cos (x-a)^{\circ}$
Hence, solve $3 \cos x^{\circ}+4 \sin x^{\circ}=5$

## 151

$f(x)=8 x^{2}-5$ and $g(x)=5+x$
Find $f(g(x))$ and $g(f(x))$.
152
The diagram shows the graph of a function $y=f(x)$.
Sketch the graphs of:
$y=f(x-4)$ and $y=2+f(x-4)$.


153
$A(0,-3,5), B(7,-6,9)$ and $C(21,-12,17)$.
Show that $A, B$ and $C$ are collinear, stating the ratio
$A B: B C$.
154
is the point $(-1,2,-1)$ and $Q$ is $(3,2,-4)$. Write down PQ in component form. Calculate the length of $\overrightarrow{P Q}$.
Find the components of a unit vector which is parallel to $\overrightarrow{P Q}$.
155
Prove the identity:

$$
\cos ^{2} Q \tan ^{2} Q=1-\cos ^{2} Q
$$

156 The point A has coordinates (7, 4). The straight
lines with equations $x+3 y+1=0$ and
$2 x+5 y=0$ intersect at B.
Find the gradient of $A B$.
157
A triangle has vertices $A(5,5), B(-10,0)$ and $C(0,-10)$. Find the equation of the altitude from A.

158
A circle has centre $\mathrm{C}(-2,3)$ and passes through $\mathrm{P}(1,6)$. Find the equation of the circle.

## 159

A sequence is defined by the recurrence relation
$u_{n+1}=0.8 u_{n}+12, u_{0}=4$.
State why this sequence has a limit and find this limit.

## 160

Calculate the area between the line $y=x+y=18$ and the curve $y=x^{2}-8 x+18$.

$161_{\text {Show that }}(x+2)$ is a factor of $f(x)=x^{3}-2 x^{2}-4 x+8$ and hence fully factorise $f(x)$.
162
Calculate the discriminant of the quadratic equation $2 x^{2}+4 x+5=0$

## 163

Solve the equation $\log _{4}(5-x)-\log _{4}(3-x)=2$, $x<3$.

## 164

Find all the values of $x$ in the interval $0 \leq x \leq 2 \pi$ for which $\tan ^{2}(x)=3$.

## 165

Show that the exact value of $\cos 2 x$ is $\frac{7}{25}$.


166
If $y=3 \cos ^{4} x$, find $\frac{d y}{d x}$.

167
Á curve has equation $y=x^{3}-3 x^{2}-9 x+12$.
Find the coordinates of the stationary points of this curve and determine their nature.

168
Find $\int \frac{4 x^{3}-1}{x^{2}} d x, x \neq 0$.

169
Find the value of $\int_{0}^{2} \sin (4 x+1) d x$.

170
A curve has equation $y=7 \sin x-24 \cos x$.
Express $7 \sin x-24 \cos x$ in the form $k \sin (x-a)$
where $k>0$ and $0 \leq a \leq \frac{\pi}{2}$.

171
$f(x)=3 x-1$ and $g(x)=\frac{1}{x+1}$
Find $f(g(x))$ and $g(f(x))$.
State a suitable domain for $g(f(x))$.
172
The diagram shows the graph $y=g(x)$.
a. Sketch $y=-g(x)$
b. Sketch $y=3-g(x)$


173
If $\boldsymbol{f}=3 \boldsymbol{i}+2 \boldsymbol{k}$ and $\boldsymbol{g}=2 \boldsymbol{i}+4 \boldsymbol{j}+3 \boldsymbol{k}$,
Find $|\boldsymbol{f}+\boldsymbol{g}|$.

## 174

Express the vectors $\overrightarrow{T A}$ and $\overrightarrow{T B}$ in component form.
Calculate the angle between $\overrightarrow{T A}$ and $\overrightarrow{T B}$.
175
Prove the identity:

$$
\left(\cos P^{\circ}+\sin P^{\circ}\right)^{2}=2 \sin P^{\circ} \cos P^{\circ}+1
$$

## 176

Find the equation of the line $S T$, where $T$ is the point $(-2,0)$ and angle STO is $60^{\circ}$.


177
Triangle $A B C$ has vertices $A(-1,12), B(-2,-5)$
and $C(7,-2)$. Find the equation of the altitude AE .

178
Show that the line with equation $y=6-2 x$ in a tangent to the circle with equation $x^{2}+y^{2}+6 x-4 y-7=0$ and find the coordinates of the point of contact of the tangent and the circle.

179
A sequence is defined by the recurrence relation
$u_{n+1}=0.2 u_{n}+5$ with $u_{8}=20$.
Calculate $u_{10}$.

180
Calculate the area enclosed between the curve $y=x^{2}-6 x$ and the $x$-axis.


## 181

Show that $(3 x+1)$ is a factor of $g(x)=3 x^{3}+4 x^{2}-5 x-2$.
Hence fully factorise $g(x)$.
182
Solve $1-2 x-3 x^{2}>0$, where $x$ is a real number.

## 183

Solve the equation $\log _{2}(x+1)-2 \log _{2} 3=3$.

## 184

Solve $2 \tan 3 x+2=0$ for $0 \leq x \leq 360$.

185 A right-angled triangle has sides and angles as shown in the diagram. What is the value of $\sin 2 a$ ?


186
Given that $y=\sin \left(x^{2}-3\right)$, find $\frac{d y}{d x}$.

187
A curve has equation $y=3 x^{2}-x^{3}$. Find the coordinates of the stationary points on this curve and determine their nature.

188
Find $\int(2 x+9)^{5} d x$

189
Find $\int_{0}^{2} \sqrt{4 x+1} d x$.

190
Express $f(x)=\sqrt{3} \cos x+\sin x$ in the form
$k \sin (x+a)$, where $k>0$ and $0<a<\frac{\pi}{2}$.

191
A function $f$, defined on a suitable domain, is
given by $f(x)=\frac{6 x}{x^{2}+6 x-16}$.
What restrictions are there on the domain of $f$ ?
192
The diagram shows part of the graph of $y=f(x)$. Sketch the graph of $y=2 f(x)+1$


193
$\boldsymbol{p}=-\boldsymbol{i}+3 \boldsymbol{j}+4 \boldsymbol{k}$ and $\boldsymbol{q}=7 \boldsymbol{i}-\boldsymbol{j}+5 \boldsymbol{k}$
a) Express $\overrightarrow{P Q}$ in component form.
b) Find the length of $P Q$.

## 194

The vectors $\boldsymbol{u}=\left(\begin{array}{l}1 \\ k \\ k\end{array}\right)$ and $\boldsymbol{v}=\left(\begin{array}{c}-6 \\ 2 \\ 5\end{array}\right)$ are perpendicular.
What is the value of k ?

## 195

Show that:
$(1+2 \sin x)(1-2 \sin x)=4 \cos ^{2} x-3$

196
Find the equation of the line through the point $(-1,4)$ which is parallel to the line with equation $3 x-y+2=0$.

## 197

A triangle has vertices $P(-2,2), Q(6,6)$ and $R(6,-4)$
Find the equation of the perpendicular bisector of PR.

198
Find $P$ and $Q$, the points of intersection of the
line $y=3 x-5$ and the circle $C_{1}$ with equation
$x^{2}+y^{2}+2 x-4 y-15=0$.
199
A sequence is defined by the recurrence relation
$u_{n+1}=\frac{1}{4} u_{n}+16, u_{0}=0$.
Calculate the values of $u_{1}, u_{2}$, and $u_{3}$.

## 200

Calculate the shaded area between the curve $y=-x^{2}+7 x-10$ and the $x$-axis.
$\xrightarrow{\stackrel{y}{4}}$

| Ques 1-20 | Ques 21-40 |
| :---: | :---: |
| 1. $f(x)=(x-1)(2 x+5)(x-1)$ | 21. $f(x)=(x-1)(x+2)(x-1)$ |
| 2. $(x+4)^{2}-13 \quad$ Min T.P at $(-4,-13)$ | 22. $q=5$ |
| 3. 2 | 23. $x=2$ |
| 4. $x=\frac{2 \pi}{3}$ for $\frac{\pi}{2} \leq x \leq \pi$ | 24. $x=\frac{\pi}{6}, \frac{11 \pi}{6}$ |
| 5. $\sin (x+a)=\frac{4}{5} \sin x+\frac{3}{5} \cos x$ | $\text { 25. } \cos 2 x=\frac{-3}{5}$ |
| 6. $\frac{d y}{d x}=12 x^{2}+10 x-3$ | 26. $f^{\prime}(x)=3 x\left(4-3 x^{2}\right)^{\frac{-3}{2}}$ |
| 7. Max T.P at $(-1,17)$ and Min T.P. at $(3,-15)$ | 27. Max T.P. at (-1,4) and Min T.P. at (1, 0) |
| 8. $\frac{-2 x^{-3}}{3}+\frac{1}{5} \sin 5 x+C$ | 28. $\frac{8}{3} x^{\frac{3}{2}}-\frac{1}{2} x^{-2}+C$ |
| 9. $y=4 x^{2}-3 x-3$ | 29. $y=\frac{-1}{3} \cos 3 x+\frac{7}{6}$ |
| 10. $k=2$ and $a=30^{\circ}$ |  |
| 11. $x \leq 3$ | 30. $\sqrt{2} \sin \left(x-\frac{\pi}{4}\right)$ |
| 12. $a=2 \quad b=3$ | 31. $f(g(x))=3 x^{2}-5 g(f(x))=9 x^{2}+6 x-1$ |
| 13. 1:2 | 32. $a=3 \quad b=3$ |
| 14. 26 | 33. $k=4$ |
| 15. $\sin 60=\frac{\sqrt{3}}{2} \tan \frac{\pi}{6}=\frac{1}{\sqrt{3}}$ | 34. $\overrightarrow{D E}=3 \overrightarrow{E F}$ so $\overrightarrow{D E}$ and $\overrightarrow{E F}$ are parallel. E is a common point so D,E,F are collinear. |
| 16. $m=-1$ | 35. Proof. |
| 17. $y=-3 x+10$ | 36. $3 y+5 x=-13$ |
| 18. $(x+7)^{2}+(y-6)^{2}=36$ | 37. $m_{P S}=\frac{7}{4}$ |
| 19. $u_{12}=8.7$ | 38. $4 y+5 x=71$ |
| 20. Area $=32$ square units | 39. $L=-400$ <br> 40. Area $=12 \frac{3}{20}$ square units. |
| Ques 41-60 | Ques 61-80 |
| 41. $Y=3(x-1)(x-4)$ | 61. $(x-1)(x+4)(x+5)$ |
| 42. $x>3 \quad x<-2$ | 62. $k=\frac{9}{8}$ |
| 43. $\log _{a} 5$ | $\text { 63. }-4$ |
| 44. $x=0^{\circ}, 60^{\circ}, 300^{\circ}, 360^{\circ}$ | 64. $x=60^{\circ}, 132^{\circ}, 228^{\circ}, 300^{\circ}$ |
| 45. $\sin (p+q)=\frac{2+2 \sqrt{5}}{3 \sqrt{5}}$ | 65. $\cos 2 a=\frac{7}{25}$ |
| 46. $\frac{d y}{d x}=6 x^{5}+24 x^{2}$ | 66.14 ${ }^{\text {67 }}$ |
| 47. $y=6 x-18$ | 67. $y=-x+1$ |
| 48. $-2 \cos (2 x+3)+C$ | 68. $\frac{x^{-3}}{-9}+C$ |
| 49. $9 \frac{1}{3}$ | 69.1 |
| 50. $\sqrt{13} \sin (x-303.7)$ | $70.5 \cos \left(x+306.9^{\circ}\right)$ |
| 51. 0.5 | $\text { 71. } f(g(x))=x^{2}+8 x+19 g(f(x))=x^{2}+7$ $\text { 72. } q=13$ |
| 52. Correct shape, Min T.P at (-4, -4) Max T.P. at ( $-1,1$ ) | 73. 3:2 |
| 53. $\left(\begin{array}{c}8 \\ -4 \\ -5\end{array}\right)$ | 74. $\frac{9}{2}$ |
| 54, $x$-5 | 75. $120^{\circ}=\frac{3 \pi}{4}$ and $\frac{2 \pi}{3}=120^{\circ}$ |
| 54. $x=1$ | 76. 5 units |
| $\begin{aligned} & \text { 55. Proot } \\ & 56 m=1 \end{aligned}$ | 77. $y=4 x+4$ |
| 56. $m=\frac{1}{\sqrt{3}}$ | 78. $J(-1,-2) \quad K(1,2)$ |
| 57. $3 y-x-8=0$ | 79. $l=100 / 3$ |
| 58. Centre $(-4,-2)$ radius $=\sqrt{58}$ <br> 59. $u_{3}=29$ | 80. Area $=\frac{27}{4}$ square units |


| 60. Area $=57 \frac{1}{6}$ square units |  |
| :---: | :---: |
| Ques 81-100 | Ques 101-120 |
| 81. $K=-2$ and $t=-5$ <br> 82. $x>0 \quad x<-4$ <br> 83. $x=\frac{3}{8}$ <br> 84. $x=30^{\circ}, 90^{\circ}, 150^{\circ}$ <br> 85. $\sin (a+b)=\frac{63}{65}$ <br> 86. $f^{\prime}(x)=-\frac{1}{5} x^{-\frac{6}{5}}$ <br> 87. $y=4 x-2$ <br> 88. $\frac{(2 x-1)^{\frac{3}{2}}}{3}+C$ <br> 89. $\frac{2}{3}$ <br> 90. $\sqrt{34} \cos (x-59.0)$ 。 <br> 91. $f(g(x))=9 x^{2}-24 x+17 g(f(x))=3 x^{2}-1$ <br> 92. $a=4 b=2 c=1$ <br> 93. 3:2 <br> 94. $\theta=72^{\circ}$ <br> 95. - 1 <br> 96. $y=\frac{1}{4} x+\frac{13}{4}$ <br> 97. $y=\frac{3}{2} x-3$ <br> 98. Centre $(-1,-2)$ Radius $\sqrt{32}$ <br> 99. $L=\frac{28}{3}$ <br> 100 Area $=9$ square units | $1014$ $102 . q=-9$ $103 x=2 y^{3}$ $104 . x=20^{\circ}, 100^{\circ}, 140^{\circ}, 220^{\circ}, 260^{\circ}, 340^{\circ}$ $105 \cdot \sin 2 x=\frac{4}{5}$ <br> 106. 1 <br> 107Max T.P. when $x=\frac{1}{3}$ $\text { 108. } x^{3}+x^{2}+c$ $\text { 109. } f(x)=2 x+\frac{1}{x}+5$ <br> $110.2 .5 \cos (x+306.9){ }^{\circ}$ $111 x \neq 3$ and $x \neq 4$ <br> 111. $x \neq 3$ and $x \neq 4$ <br> 112 Correct shape drawn and labelled with $(0,3),(3,1),(5,3)$ $113 . Q(3,1,-2)$ <br> $114 . t=-3$ <br> 115.Proof. $116 . m=1$ $\text { 117. } y=-2 x+2$ $118 . y=-\frac{3}{2} x+6$ <br> $119 . u_{4}=3$ <br> 120. $9 / 8$ |
| Ques 121-140 | Ques 141-160 |
| 121. $y=-x(x+1)(x-2)$ <br> 122. $x<-5$ and $x>3$ <br> 123.1 <br> 124. $x=60^{\circ}, 120^{\circ}, 240^{\circ}, 300^{\circ}$ <br> 125.Proof. <br> 126.12 <br> 127. $y=2 x-12$ <br> 128. $-\frac{(1-6 x)^{\frac{1}{2}}}{3}+C$ <br> 129. $y=2 x^{3}-2 x^{2}+3 x+2$ <br> 130. $10 \cos (x+36.9)^{\circ}$ <br> 131. $p(x)=3-\frac{3}{x}$ and $p(q(x))=x$ <br> 132. $y=-2 f(x)$ passing through $(-6,0),(1,14),(3,0)$ and $y=f(x-3)$ passing through $(-3,0),(4,-7),(6,0)$ <br> 133. $\overrightarrow{Q R}=2 \overrightarrow{P Q}$ and Q is a common point so $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ are collinear. <br> 134. 5 <br> 135. Proof. $\begin{aligned} & \text { 136. } y=-\frac{1}{2} x+\frac{13}{2} \\ & \text { 137. } y=6 x+9 \\ & \text { 138. } y=\frac{1}{4} x-\frac{7}{2} \\ & \text { 139. } L=-50 \\ & \text { 140. Area }=\frac{20}{3} \text { square units } \end{aligned}$ | $\begin{aligned} & \text { 141. }(x-4)(x-2)(x+1) \\ & \text { 142. } x<-3 \text { and } x>2 \\ & \text { 143. } k=0.46 \\ & \text { 144. } x=45^{\circ}, 105^{\circ}, 225^{\circ}, 285^{\circ} \\ & \text { 145.Proof. } \\ & \text { 146. } \frac{d y}{d x}=-6 x^{-3}+3 x^{\frac{1}{2}} \\ & \text { 147. } y=2 x-11 \\ & \text { 148. } \frac{x^{3}}{3}+4 x^{-1}+C \\ & \text { 149. } y=2 x^{2}-2 x^{3}+5 \\ & \text { 150. } x=53 \cdot 1^{\circ}, 413.1^{\circ} \\ & \text { 151. } f(g(x))=8 x^{2}+80 x+195 g(f(x))=8 x^{2} \\ & \text { 152. } y=f(x-4) \text { passing through }(0,5),(5, a) y=2+ \\ & f(x-4) \text { passing through }(0,7),(5, a+2) \\ & \text { 153.AB:BC }=1: 2 \\ & \\ & \text { 154.Unit vector }=\left(\begin{array}{l} \frac{4}{5} \\ 0 \\ \frac{-3}{5} \end{array}\right) \\ & \text { 155.Proof. } \\ & \text { 156. } m=3 \\ & \text { 157. } y=x \\ & \text { 158. }(x+2)^{2}+(y-3)^{2}=18 \\ & \text { 159. } L=60 \\ & \text { 160.Area }=\frac{343}{6} \text { square units } \end{aligned}$ |


|  |  |
| :---: | :---: |
| Ques 161-180 | Ques 181-200 |
| $\begin{aligned} & \text { 161. }(x+2)(x-2)(x-2) \\ & \text { 162.b } b^{2}-4 a c=-24 \\ & 163 . x=\frac{43}{15} \\ & \text { 164. } x=\frac{\pi}{3}, \frac{2 \pi}{3}, \frac{4 \pi}{3}, \frac{5 \pi}{3} \\ & \text { 165.Proof. } \\ & \text { 166. } \frac{d y}{d x}=-12 \sin x \cos ^{3} x \\ & \text { 167.Max T.P at }(-1,17) . \text { Min T.P at }(3,-15) . \\ & \text { 168.2 } x^{2}+\frac{1}{x}+C \\ & \text { 169.0.363 radians } \\ & \text { 170.25sin }(x-1.287) \text { (in radians) } \\ & \text { 171. } f(g(x))=\frac{-x+2}{x+1} \quad g(f(x))=\frac{1}{3 x} \\ & \text { 172. } y=-g(x) \text { passes through }(a,-2),(0,-1),(b,-3) \\ & \quad y=3-g(x) \text { passes through }(a, 5),(0,2),(b, 0) \\ & \text { 173. }\|f+g\|=\sqrt{66} \\ & \text { 174. } \theta=50.9 \\ & \text { 175.Proof. } \\ & \text { 176. } y=\sqrt{3} x+2 \sqrt{3} \\ & \text { 177. } y=-3 x+9 \\ & \text { 178.Point of contact is }(1,4) \\ & \text { 179. } u 9=9 \quad u_{10}=6.8 \\ & \text { 180.Area }=36 \text { square units } \end{aligned}$ | $\begin{aligned} & \text { 181.3 }(3 x+1)(x+2)(x-1) \\ & \text { 182. }-1<x<\frac{1}{3} \\ & \text { 183. } x=71 \\ & \text { 184. } x=45^{\circ}, 105^{\circ}, 165^{\circ}, 225^{\circ}, 285^{\circ}, 345^{\circ} \\ & \text { 185.sin } 2 a=\frac{15}{17} \\ & \text { 186. } \frac{d y}{d x}=2 x \cos \left(x^{2}-3\right) \\ & \text { 187.Min T.P at }(0,0) \text { Max T.P at }(2,4) \\ & \text { 188. } \frac{(2 x+9)^{6}}{12}+C \\ & \text { 189. } \frac{13}{3} \\ & \text { 190.2sin }\left(x+\frac{\pi}{3}\right) \\ & \text { 191. } x \neq-8, x \neq 2 \\ & \text { 192. } y=2 f(x)+1 \text { passes through }(0,1),(2,7),(5,1) \\ & \text { 193. } \overrightarrow{P Q}=\left(\begin{array}{c} 8 \\ -4 \\ 1 \end{array}\right)\|\overrightarrow{P Q}\|=9 \\ & \text { 194. } k=\frac{6}{7} \\ & \text { 195.Proof. } \\ & \text { 196. } y=3 x+7 \\ & \text { 197. } 3 y=4 x-11 \\ & \text { 198. } P(1,-2) Q(3,4) \\ & \text { 199. } u_{1}=16 \quad u_{2}=20 \quad u_{3}=21 \\ & \text { 100. Area }=\frac{9}{2} \text { square units } \end{aligned}$ |

