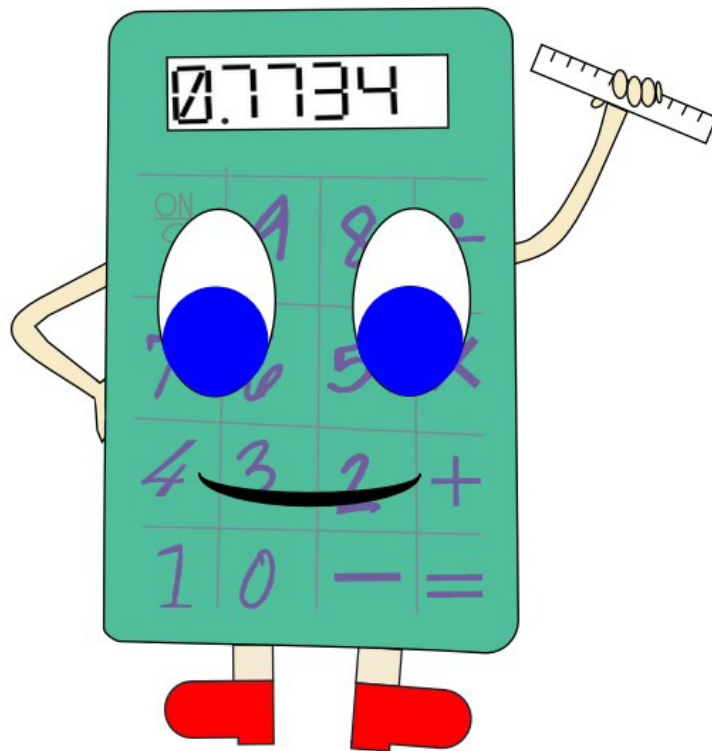


St Thomas of Aquin's RC High School



Numeracy Booklet

A guide for pupils, parents and staff

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils and parents on how certain common Numeracy topics are taught in mathematics and throughout the school. Staff from all departments have been issued with a copy of the booklet, and it is hoped that with a consistent approach across all subjects pupils will progress successfully.

How can it be used?

If you are helping your child with their Home Study, you can refer to the booklet to see what methods are being taught in school. Look up the relevant page for a step by step guide. Pupils should carry this booklet with them in school to help them solve number and information handling questions in any subject.

The booklet includes Numeracy skills useful in subjects other than mathematics. There is also a useful Mathematical Words Dictionary for reference at the back.

Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

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Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate $54 + 27$

Method 1 Add tens, then add units, then add together

$$50 + 20 = 70 \qquad 4 + 7 = 11 \qquad 70 + 11 = 81$$

Method 2 Split up the **number to be added** into tens and units and add separately.

$$54 + 20 = 74 \qquad 74 + 7 = 81$$

Method 3 Round up to nearest 10, then subtract

$$54 + 30 = 84 \quad \text{but } 30 \text{ is } 3 \text{ too much so subtract } 3;$$
$$84 - 3 = 81$$

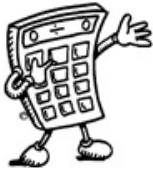
Written Method

When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

| | | | | | | |
|---|---|--|---|---|---|--|
| $\begin{array}{r} 3032 \\ +589 \\ \hline 1 \\ \hline \end{array}$ | → | $\begin{array}{r} 3032 \\ +589 \\ \hline 21 \\ \hline \end{array}$ | → | $\begin{array}{r} 3032 \\ +589 \\ \hline 621 \\ \hline \end{array}$ | → | $\begin{array}{r} 3032 \\ +589 \\ \hline 3621 \\ \hline \end{array}$ |
| | | | | | | |

Subtraction



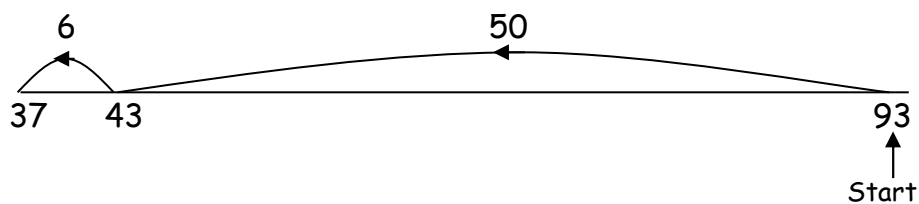
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate $93 - 56$

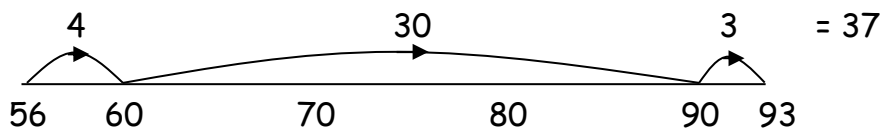
Method 1 Break up the number being subtracted

e.g. subtract 50, then subtract 6 $93 - 50 = 43$
 $43 - 6 = 37$



Method 2 Count on

Count on from 56 until you reach 93. This can be done in several ways e.g.



Written Method

Example 1 $4590 - 386$

$$\begin{array}{r} 81 \\ 4590 \\ - 386 \\ \hline 4204 \end{array}$$

Example 2 Subtract 692 from 14597

We do not "borrow and pay back".

$$\begin{array}{r} 31 \\ 14597 \\ - 692 \\ \hline 13905 \end{array}$$

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Mental Strategies

Example Find 39×6

Method 1

$$\begin{array}{r} 30 \times 6 \\ = 180 \end{array}$$

$$\begin{array}{r} 9 \times 6 \\ = 54 \end{array}$$

$$\begin{array}{r} 180 + 54 \\ = 234 \end{array}$$

Method 2

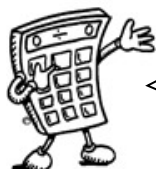
$$\begin{array}{r} 40 \times 6 \\ = 240 \end{array}$$

40 is 1 too many
so take away 6×1

$$\begin{array}{r} 240 - 6 \\ = 234 \end{array}$$

Multiplication 2

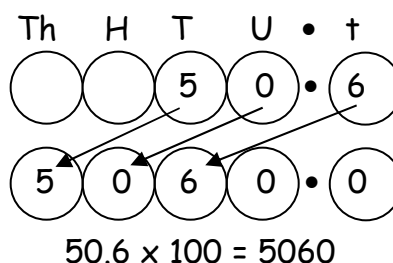
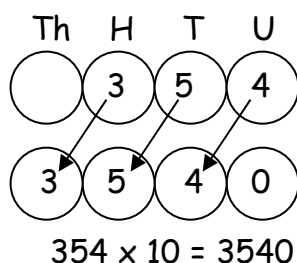
Multiplying by multiples of 10 and 100



To multiply by **10** you move every digit *one* place to the left.

To multiply by **100** you move every digit *two* places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



(c) 35×30

To multiply by 30,
multiply by 3,
then by 10.

$$35 \times 3 = 105$$

$$105 \times 10 = 1050$$

so $35 \times 30 = 1050$

(d) 436×600

To multiply by
600, multiply by 6,
then by 100.

$$436 \times 6 = 2616$$

$$2616 \times 100 = 261600$$

so $436 \times 600 = 261600$



We may also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36×20 (b) 38.4×50

$$2.36 \times 2 = 4.72$$

$$4.72 \times 10 = 47.2$$

so $2.36 \times 20 = 47.2$

$$38.4 \times 5 = 192.0$$

$$192.0 \times 10 = 1920$$

so $38.4 \times 50 = 1920$

Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 24 \\ 8 \overline{) 192} \end{array}$$

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1.58 \\ 3 \overline{) 4.74} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 3 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

$$\begin{array}{r} 0.275 \\ 8 \overline{) 2.260} \end{array}$$

Each glass contains
0.275 litres

If you have a remainder at the end of a calculation, write a zero at the end of the decimal and continue with the calculation. This continues until no remainder is achieved.

Order of Calculation (BODMAS)

Consider this: What is the answer to $2 + 5 \times 8$?

Is it $7 \times 8 = 56$ or $2 + 40 = 42$?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS**

The **BODMAS** rule tells us which operations should be done first.

BODMAS represents:

(B)rackets

(O)f

(D)ivide

(M)ultiply

(A)dd

(S)ubtract

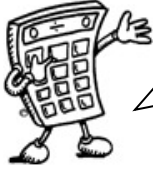
Scientific calculators use this rule, some basic calculators may not, so take care in their use.

Example 1 $15 - 12 \div 6$ BODMAS tells us to divide first
= $15 - 2$
= 13

Example 2 $(9 + 5) \times 6$ BODMAS tells us to work out the
= 14×6 brackets first
= 84

Example 3 $18 + 6 \div (5-2)$ Brackets first
= $18 + 6 \div 3$ Then divide
= $18 + 2$ Now add
= 20

Evaluating Formulae / Substitution



To find the value of a variable in a formula, we must substitute all of the given values into the formula, then use BODMAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$$P = 2L + 2B$$

$$P = 2 \times 12 + 2 \times 7$$

$$P = 24 + 14$$

$$P = 38$$

Step 1: write formula

Step 2: substitute numbers for letters

Step 3: start to evaluate (BODMAS)

Step 4: write answer

Example 2

Use the formula $I = \frac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$

$$I = \frac{240}{40}$$

$$I = 6$$

Example 3

Use the formula $F = 32 + 1.8C$ to evaluate F when $C = 20$

$$F = 32 + 1.8C$$

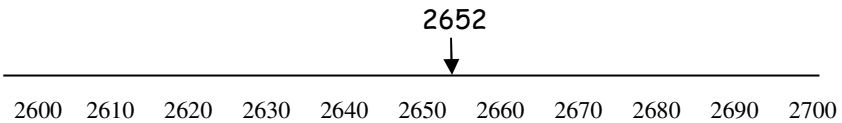
$$F = 32 + 1.8 \times 20$$

$$F = 32 + 36$$

$$F = 68$$

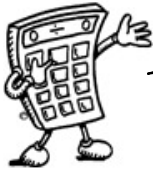
Estimation : Rounding

Numbers can be rounded to give an approximation.



2652 rounded to the nearest 10 is 2650.

2652 rounded to the nearest 100 is 2700.



When rounding numbers which are exactly in the middle, the convention is to **round up**.
7865 rounded to the nearest 10 is 7870.

The same principles apply to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right - if it is 5 or more round up.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the next digit (in the hundreds column) is a 7, so round up.

46 753
= 47 000 to the nearest thousand

Example 2 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the next digit (the third number after the decimal point) is a 3, so round down.

1.57359
= 1.57 to 2 decimal places

Estimation : Calculation



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1

Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

| Monday | Tuesday | Wednesday | Thursday |
|--------|---------|-----------|----------|
| 486 | 205 | 197 | 321 |

$$\text{Estimate} = 500 + 200 + 200 + 300 = 1200$$

$$\begin{array}{r} \text{Calculate: } 486 \\ 205 \\ 197 \\ +321 \\ \hline 1209 \end{array} \quad \text{Answer} = 1209 \text{ tickets}$$

Example 2

A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

$$\text{Estimate} = 50 \times 40 = 2000\text{g}$$

$$\text{Calculate: } 42 \times 48 = 2016\text{g}$$

Time 1

Time may be expressed in 12 or 24 hour notation.



12-hour clock

Time can be displayed on an analogue clock face, or digital clock.



05:15

These clocks both show quarter past five.

When writing times in 12 hour notation, we need to add a.m. or p.m. after the time.

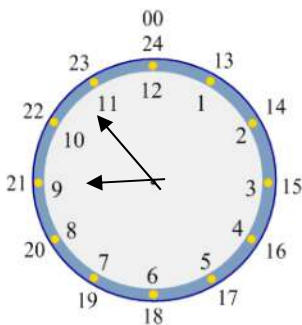
a.m. is used for times between midnight and 12 noon (morning)

p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 ... etc.



Examples

9.55 am → 09 55

3.35 pm → 15 35

12.20 am → 00 20

02 16 hours → 2.16 am

20 45 hours → 8.45 pm

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
 52 weeks
 12 months

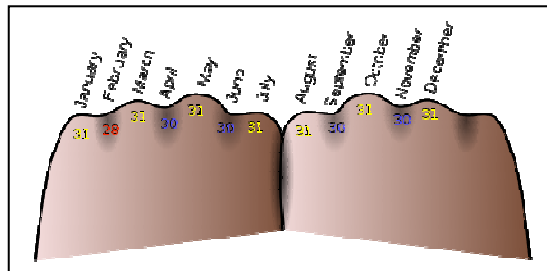
A **decade** is 10 years.

A **century** is 100 years.

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

There is also an easy way to remember the days in a month using your knuckles.



Put your hands together leaving out your thumb knuckle as shown above. Begin counting through the months from your furthest left knuckle, counting in turn the knuckles and the grooves in between.

Rule: Every month which lands on a knuckle has 31 days.
Every month which lands on a groove has 30 days
(except February 28 days or 29 in leap year)

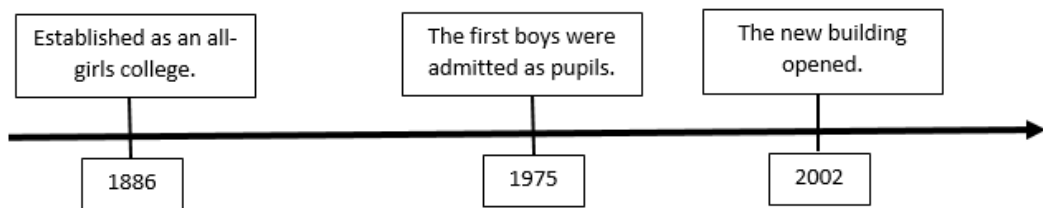
Time 3

Timelines

A timeline represents a period of time, on which important events are marked.

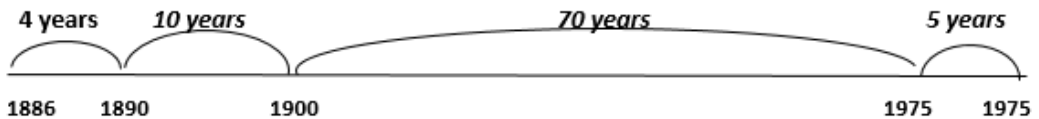
Example 1

Below is a timeline of our school:



Example 2

For how many years was St Thomas of Aquin's an all-girls College?



$$4 + 10 + 70 + 5 = \underline{\underline{89 \text{ years}}}$$

Look back at 'subtraction' for other possible methods.

Important Information

B.C → Before Christ

A.D → Anno Domini (*in the year of our Lord*)

Time 4

Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

$$\text{Distance} = \text{Speed} \times \text{Time} \quad \text{or} \quad D = S T$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$

Example One Calculate the speed of a train which travelled

$$\begin{aligned} 450 \text{ km in 5 hours } S &= \frac{D}{T} \\ S &= \frac{450}{5} \\ S &= 90 \text{ km/h} \end{aligned}$$

Example Two Calculate the distance travelled at a speed of 15km/h for 3 and a half hours.

$$\begin{aligned} D &= S T \\ D &= 15 \times 3.5 \\ D &= 52.5 \text{ km} \end{aligned}$$

Example Three Calculate the time it takes for Kathryn to walk to school, a distance of 5km, at a speed of 4 km/h.

$$\begin{aligned} T &= \frac{D}{S} \\ T &= \frac{5}{4} = 1.25 \text{ h} \\ &= 1 \text{ hour } 15 \text{ minutes} \end{aligned}$$

Important Note In these formulae time must be written as a decimal fraction of an hour.

To convert a number of minutes into a decimal fraction divide by 60.

To convert a decimal fraction of an hour into minutes multiply by 60.

Fractions 1

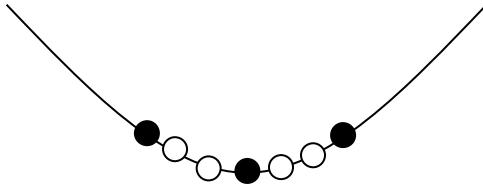


Addition, subtraction, multiplication and division of fractions are studied in mathematics. However, the examples below may be helpful in all subjects.

Understanding Fractions

Example

A necklace is made from black and white beads.



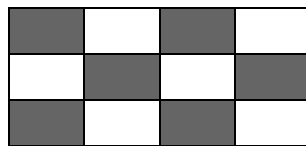
What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example

What fraction of the flag is shaded?



6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.

To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

(a) $\frac{20}{25} \xrightarrow{\div 5} \frac{4}{5}$

(b) $\frac{16}{24} \xrightarrow{\div 8} \frac{2}{3}$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in its **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find a fraction of a quantity, divide by the denominator, then multiply the answer by the numerator.

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{3}{7}$ divide by 7, then multiply by 3 etc.

Example 1

Find $\frac{1}{5}$ of £150

$$\begin{aligned} \frac{1}{5} \text{ of } \pounds 150 &= 150 \div 5 \\ &= \pounds 30 \end{aligned}$$

Example 2

Find $\frac{3}{4}$ of 48

$$\begin{aligned} \frac{3}{4} \text{ of } 48 &= 48 \div 4 \times 3 \\ &= 12 \times 3 \\ &= 36 \end{aligned}$$

Percentages 1



Percent means out of 100.

A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$

36% is therefore equivalent to $\frac{9}{25}$ and 0.36

Common Percentages

Some percentages are used very frequently. It is essential to know and recall these as fractions and decimals.

| Percentage | Fraction | Decimal |
|-------------------|-----------------|----------|
| 1% | $\frac{1}{100}$ | 0.01 |
| 10% | $\frac{1}{10}$ | 0.1 |
| 20% | $\frac{1}{5}$ | 0.2 |
| 25% | $\frac{1}{4}$ | 0.25 |
| $33\frac{1}{3}\%$ | $\frac{1}{3}$ | 0.333... |
| 50% | $\frac{1}{2}$ | 0.5 |
| $66\frac{2}{3}\%$ | $\frac{2}{3}$ | 0.666... |
| 75% | $\frac{3}{4}$ | 0.75 |

Percentages 2



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non-Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

$$25\% \text{ of } \pounds 640 = \frac{1}{4} \text{ of } \pounds 640 = \pounds 640 \div 4 = \pounds 160$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

$$1\% \text{ of } 200\text{g} = \frac{1}{100} \text{ of } 200\text{g} = 200\text{g} \div 100 = 2\text{g}$$

$$\text{so } 9\% \text{ of } 200\text{g} = 9 \times 2\text{g} = 18\text{g}$$

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

$$10\% \text{ of } \pounds 35 = \frac{1}{10} \text{ of } \pounds 35 = \pounds 35 \div 10 = \pounds 3.50$$

$$\text{so } 70\% \text{ of } \pounds 35 = 7 \times \pounds 3.50 = \pounds 24.50$$

Percentages 3

Non-Calculator Methods (continued)

The previous 2 methods can be combined to calculate any percentage.

Example Find 23% of £15000

$$10\% \text{ of } \pounds 15000 = \pounds 1500 \quad \text{so } 20\% = \pounds 1500 \times 2 = \pounds 3000$$

$$1\% \text{ of } \pounds 15000 = \pounds 150 \quad \text{so } 3\% = \pounds 150 \times 3 = \pounds 450$$

$$23\% \text{ of } \pounds 15000 = \pounds 3000 + \pounds 450 = \pounds 3450$$

Finding VAT (without a calculator)

Value Added Tax (VAT) = 20% (from 4th January 2010)

To find VAT, divide by 5.

Example Calculate the total price of a computer which costs £650 excluding VAT

$$20\% \text{ of } \pounds 650 = \frac{1}{5} \text{ of } 650$$

$$= 650 \div 5$$

$$= 130$$

$$\text{Total price} = 650 + 130$$

$$= \pounds 780$$

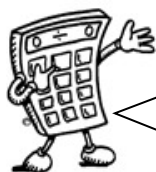
Percentages 4

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 23% of £15000

$$23\% = 0.23 \text{ so } 23\% \text{ of } \pounds 15000 = 0.23 \times \pounds 15000 = \pounds 3450$$



We do not use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

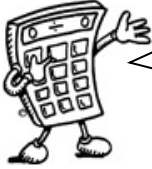
$$19\% = 0.19 \quad \text{so} \quad \text{Increase} = 0.19 \times \pounds 236000 \\ = \pounds 44840$$

$$\text{Value at end of year} = \text{original value} + \text{increase} \\ = \pounds 236000 + \pounds 44840 \\ = \pounds 280840$$

The new value of the house is £280840

Percentages 5

Finding the percentage



To find a percentage of a total, first make a fraction. Convert to a percentage by dividing the top by the bottom and multiplying by 100.

Example 1 There are 30 pupils in Class A3. 18 are girls. What percentage of Class A3 are girls?

$$\frac{18}{30} = 18 \div 30 \times 100 = 60\%$$

60% of A3 are girls

Example 2 James scored 36 out of 44 in his biology test. What is his percentage mark?

$$\text{Score} = \frac{36}{44}$$

$$36 \div 44 \times 100 = 0.81818... \times 100$$

$$= 81.818... \% = 81.82\% \text{ (to two decimal places)}$$

Example 3 In class P1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

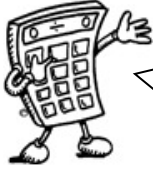
$$\text{Total number of pupils} = 14 + 6 + 3 + 2 = 25$$

6 out of 25 were blonde, so,

$$\frac{6}{25} \times 100 = 24\%$$

24% were blonde.

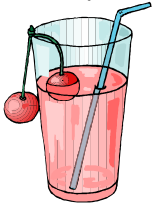
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.

The ratio of water to cordial is 4:1
(said "4 to 1")

The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2



In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

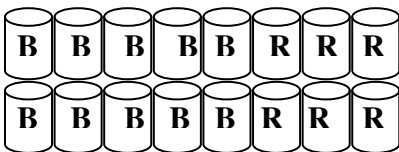
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



$$\begin{aligned} \text{Blue : Red} &= 10 : 6 \\ &= 5 : 3 \end{aligned}$$

To simplify a ratio, divide each figure by the highest common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4:6

(b) 24:36

(c) 6:3:12

(a) 4:6
= 2:3

Divide each
figure by 2

(b) 24:36
= 2:3

Divide each
figure by 12

(c) 6:3:12
= 2:1:4

Divide each
figure by 3

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg of cement. Write the ratio of sand : cement in its simplest form

$$\begin{aligned}\text{Sand : Cement} &= 20 : 4 \\ &= 5 : 1\end{aligned}$$

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

| Fruit | Nuts |
|-------|------|
| 3 | 2 |
| 15 | 10 |

So the chocolate bar will contain 10g of nuts.

Ratio 3

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$$3 + 2 = 5$$

Step 2 Divide the total amount by this number to find the value of one part

$$90 \div 5 = \text{£}18$$

Step 3 Multiply to find the value of each part

$$3 \times \text{£}18 = \text{£}54$$

$$2 \times \text{£}18 = \text{£}36$$

Step 4 Check that the total is correct

$$\text{£}54 + \text{£}36 = \text{£}90$$

Lauren received £54 and Sean received £36

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1

A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

| Days | Cars |
|------|-------------|
| 30 | 1500 |
| 90 | 4500 |

$\left. \begin{array}{l} 30 \\ 90 \end{array} \right\} \times 3$
 $\left. \begin{array}{l} 1500 \\ 4500 \end{array} \right\} \times 3$

The factory would produce 4500 cars in 90 days.

Example 2

5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

| Tickets | Cost | Working: |
|---------|--------|---|
| 5 | £27.50 | $\begin{array}{r} \text{£}5.50 \quad \text{£}5.50 \\ 5 \overline{) \text{£}27.50} \quad \underline{\quad} \\ \quad \text{4} \times 8 \\ \underline{\quad} \\ \text{£}44.00 \end{array}$ |
| 1 | £5.50 | |
| 8 | £44.00 | |

The cost of 8 tickets is £44

Information Handling : Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

| | | | | | | | | | | | | |
|-----------|----|----|----|----|----|-----------|----|----|----|----|----|----|
| | J | F | M | A | M | J | J | A | S | O | N | D |
| Barcelona | 13 | 14 | 15 | 17 | 20 | 24 | 27 | 27 | 25 | 21 | 16 | 14 |
| Edinburgh | 6 | 6 | 8 | 11 | 14 | 17 | 18 | 18 | 16 | 13 | 8 | 6 |

The average maximum temperature in June in Barcelona is 24°C

Frequency Tables are used to present information. Often data is grouped in intervals.

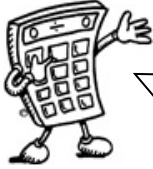
Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
 33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

| Mark | Tally | Frequency |
|---------|-------|-----------|
| 16 - 20 | | 2 |
| 21 - 25 | | 7 |
| 26 - 30 | | 9 |
| 31 - 35 | | 5 |
| 36 - 40 | | 3 |
| 41 - 45 | | 2 |
| 46 - 50 | | 2 |

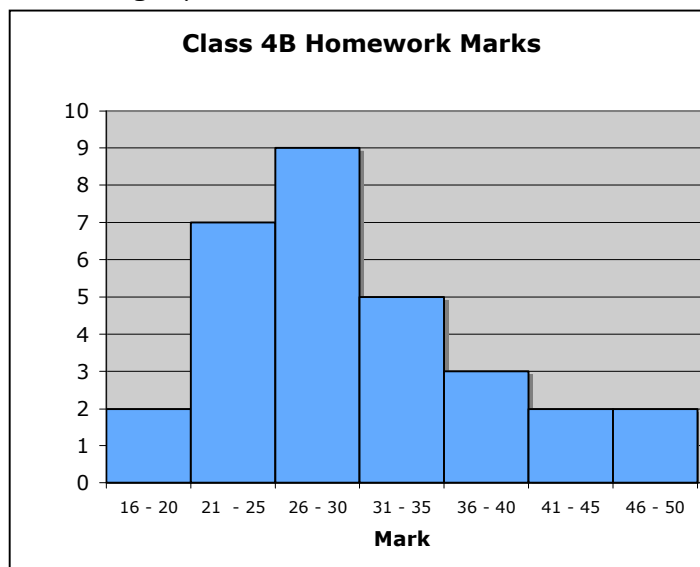
Each mark is recorded in the table by a tally mark.
 Tally marks are grouped in 5's to make them easier to read and count.

Information Handling : Bar Graphs

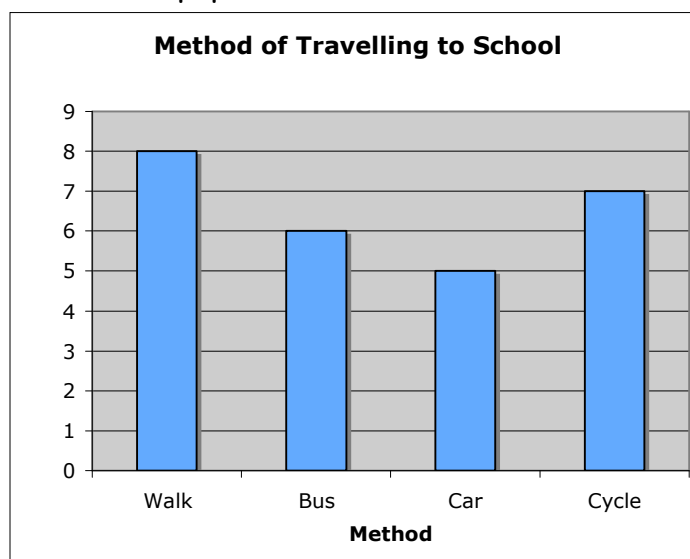


Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 The graph below shows the homework marks for Class 4B.

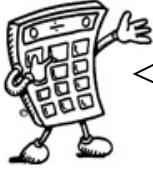


Example 2 How do pupils travel to school?



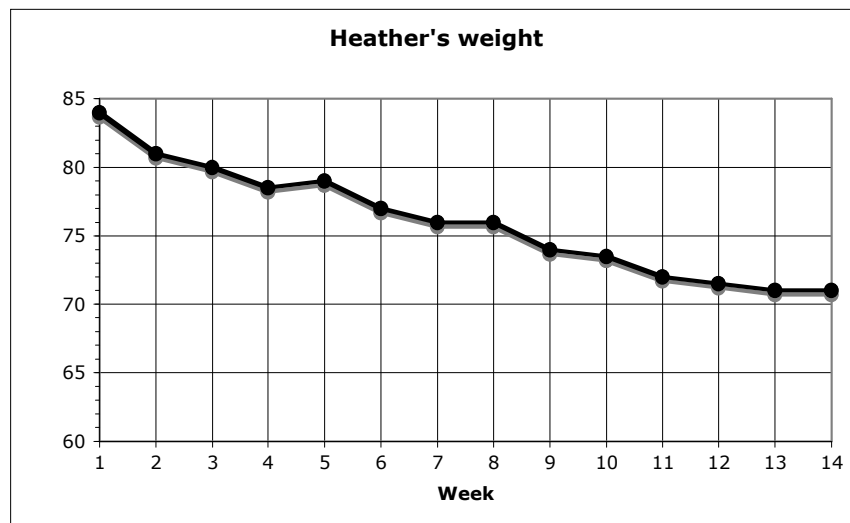
When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps between the bars.

Information Handling : Line Graphs



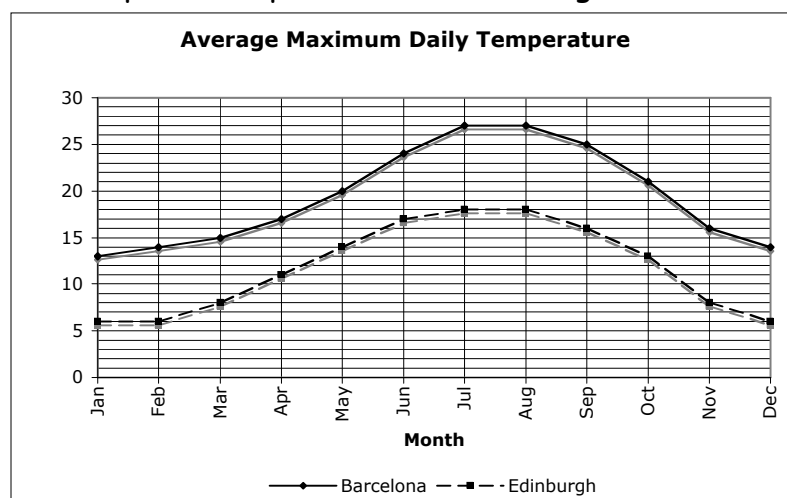
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

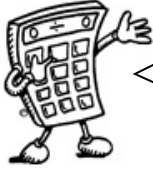


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



Information Handling : Scatter Graphs

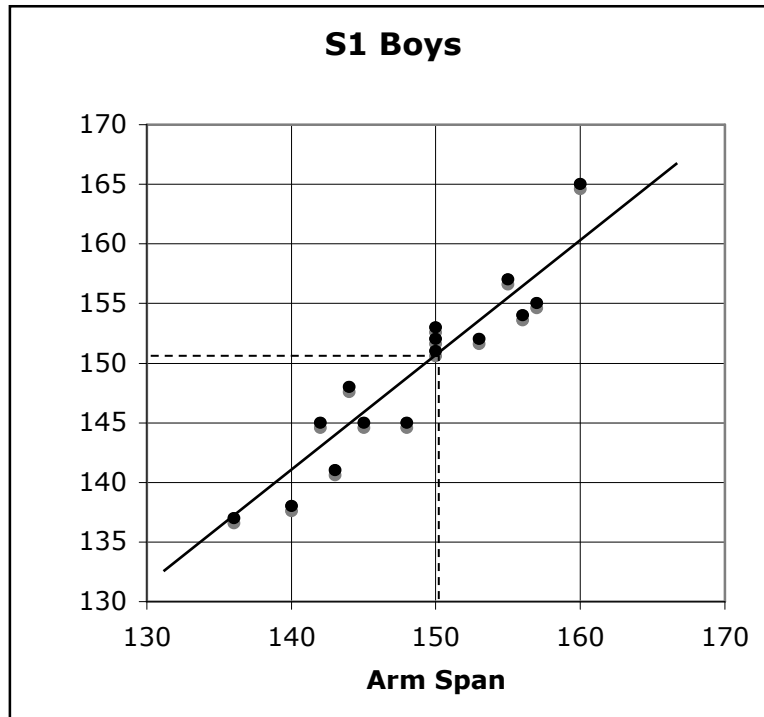


A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example

The table below shows the arm span and height of a group of first year boys. This is then plotted as a series of points on the graph below.

| | | | | | | | | | | | | | | | |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Arm Span (cm) | 150 | 157 | 155 | 142 | 153 | 143 | 140 | 145 | 144 | 150 | 148 | 160 | 150 | 156 | 136 |
| Height (cm) | 153 | 155 | 157 | 145 | 152 | 141 | 138 | 145 | 148 | 151 | 145 | 165 | 152 | 154 | 137 |



The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

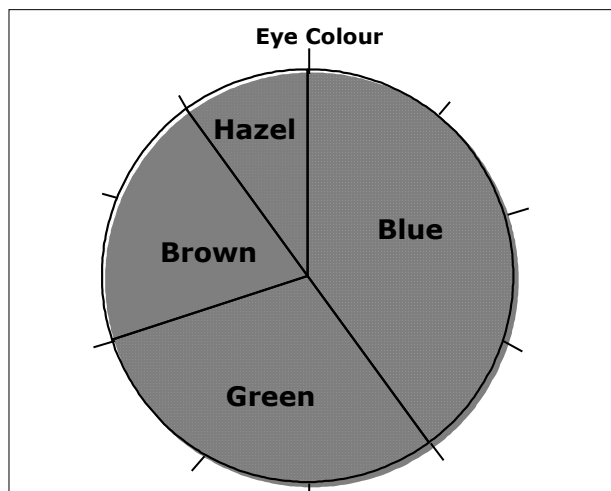
Information Handling : Pie Charts



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example

30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72° .
so the number of pupils with brown eyes
= $\frac{72}{360} \times 30 = 6$ pupils.

If finding all of the values, you can check your answers - the total should be 30 pupils.

Information Handling : Pie Charts 2

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360° .

Example: In a survey about television programmes, a group of people were asked what was their favourite soap. Their answers are given in the table below. Draw a pie chart to illustrate the information.

| Soap | Number of people |
|-------------------|------------------|
| Eastenders | 28 |
| Coronation Street | 24 |
| Emmerdale | 10 |
| Hollyoaks | 12 |
| None | 6 |

Total number of people = 80

$$\text{Eastenders} = \frac{28}{80} \rightarrow \frac{28}{80} \times 360^\circ = 126^\circ$$

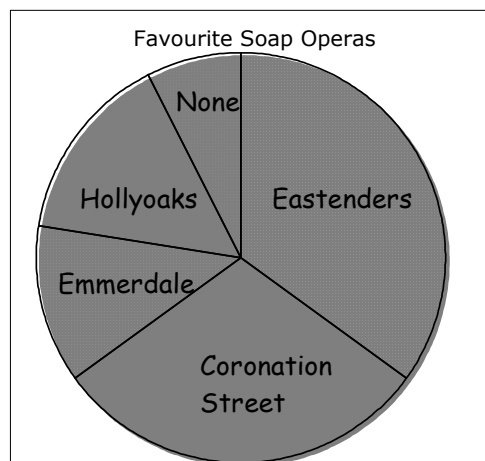
$$\text{Coronation Street} = \frac{24}{80} \rightarrow \frac{24}{80} \times 360^\circ = 108^\circ$$

$$\text{Emmerdale} = \frac{10}{80} \rightarrow \frac{10}{80} \times 360^\circ = 45^\circ$$

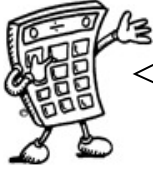
$$\text{Hollyoaks} = \frac{12}{80} \rightarrow \frac{12}{80} \times 360^\circ = 54^\circ$$

$$\text{None} = \frac{6}{80} \rightarrow \frac{6}{80} \times 360^\circ = 27^\circ$$

Check that the total = 360°



Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value - the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value - Lowest value

Example Class 4B scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 7, 10, 9, 8, 4, 8, 5, 7, 10

$$\begin{aligned}\text{Mean} &= \frac{7+9+7+5+6+7+10+9+8+4+8+5+7+10}{14} \\ &= \frac{102}{14} = 7.285\dots \quad \text{Mean} = 7.3 \text{ to 1 decimal place}\end{aligned}$$

Ordered values: 4, 5, 5, 6, 7, 7, 7, 7, 8, 8, 9, 9, 10, 10
Median = 7

7 is the most frequent mark, so Mode = 7

Range = 10 - 4 = 6

Mathematical Dictionary (Key words):

| | |
|----------------------|--|
| Add; Addition (+) | To combine 2 or more numbers to get one number (called the sum or the total) Example: $12+76 = 88$ |
| a.m. | (ante meridiem) Any time in the morning (between midnight and 12 noon). |
| Approximate | An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place. |
| Calculate | Find the answer to a problem. It doesn't mean that you must use a calculator! |
| Data | A collection of information (may include facts, numbers or measurements). |
| Denominator | The bottom number in a fraction (the number of parts into which the whole is split). |
| Difference (-) | The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$ |
| Division (\div) | Sharing a number into equal parts. $24 \div 6 = 4$ |
| Double | Multiply by 2. |
| Equals (=) | Makes or has the same amount as. |
| Equivalent fractions | Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions |
| Estimate | To make an approximate or rough answer, often by rounding. |
| Evaluate | To work out the answer. |
| Even | A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8. |
| Factor | A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15. |
| Frequency | How often something happens. In a set of data, the number of times a number or category occurs. |
| Greater than ($>$) | Is bigger or more than. Example: 10 is greater than 6. $10 > 6$ |
| Least | The lowest number in a group (minimum). |
| Less than ($<$) | Is smaller or lower than. Example: 15 is less than 21. $15 < 21$. |

| | |
|---------------------|---|
| Maximum | The largest or highest number in a group. |
| Mean | The arithmetic average of a set of numbers (see p33) |
| Median | Another type of average - the middle number of an ordered set of data (see p33) |
| Minimum | The smallest or lowest number in a group. |
| Minus (-) | To subtract. |
| Mode | Another type of average - the most frequent number or category (see p33) |
| Most | The largest or highest number in a group (maximum). |
| Multiple | A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72 |
| Multiply (x) | To combine an amount a particular number of times. Example $6 \times 4 = 24$ |
| Negative Number | A number less than zero. Shown by a minus sign. Example -5 is a negative number. |
| Numerator | The top number in a fraction. |
| Odd Number | A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9. |
| Operations | The four basic operations are addition, subtraction, multiplication and division. |
| Order of operations | The order in which operations should be done. BODMAS (see p9) |
| Place value | The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100. |
| p.m. | (post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). |
| Prime Number | A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it only has 1 factor. |
| Product | The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20. |
| Remainder | The amount left over when dividing a number. |
| Share | To divide into equal groups. |
| Square | Multiply by itself. Example 3^2 (say "3 squared") = $3 \times 3 = 9$ |
| Sum | The total of a group of numbers (found by adding). |
| Total | The sum of a group of numbers (found by adding). |