Mathematics Higher

Revision Materials

Functions, Quadratics & Polynomials Skills Builder

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

This booklet should be used to identify any areas for improvement **before** you sit the Unit assessment for the first time.

Unit	Assessment standard	Sub-skills
H4LD 76 **Relationships and Calculus**	RC1.1 Applying algebraic skills to solve equations	♦ factorising a cubic polynomial expression with unitary x^3 coefficient ♦ solving cubic polynomial equations with unitary x^3 coefficient ♦ given the nature of the roots of an equation, use the discriminant to find an unknown
	RC#2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	For candidates undertaking the Course, Assessment Standard 2.1 should be achieved on **at least two** occasions from across the Course.
H4LC 76 **Expressions and Functions**	EF1.3 Applying algebraic and trigonometric skills to functions	♦ identifying and sketching related algebraic functions ♦ identifying and sketching related trigonometric functions ♦ determining composite and inverse functions (knowledge and use of the terms domain and range is expected)
	EF#2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	For candidates undertaking the Course, Assessment Standard 2.1 should be achieved on **at least two** occasions from across the Course.
	EF#2.2 Explaining a solution and, where appropriate, relating it to context	For candidates undertaking the Course, Assessment Standard 2.2 should be achieved on **at least two** occasions from across the Course.

CfE Higher Maths. Unit Assessment Skills Builder

RC1.1 Applying algebraic skills to solve equations

RC#2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy
Sub-skills

- factorising a cubic polynomial expression with unitary x^3 coefficient (extended to non-unitary)
- solving cubic polynomial equations with unitary x^3 coefficient (extended to non-unitary)

Q1 **a)** Show that $(x-1)$ is a factor of $f(x) = 2x^3 - 5x^2 - 2x + 5$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 - x^2 - 2x + 3 = 4x^2 - 2$

Q2 **a)** Show that $(x-3)$ is a factor of $f(x) = 3x^3 - 17x^2 + 29x - 15$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $3x^3 - 15x^2 + 29x - 14 = 2x^2 + 1$

Q3 **a)** Show that $(x-1)$ is a factor of $f(x) = 2x^3 - 3x^2 - 5x + 6$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 - 2x^2 + 3x + 13 = x^2 + 8x + 7$

Q4 **a)** Show that $(x+3)$ is a factor of $f(x) = x^3 + 4x^2 - 17x - 60$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 + 4x^2 - 17x - 61 = x^3 - 1$

Q5 **a)** Show that $(x-2)$ is a factor of $f(x) = 2x^3 - 9x^2 + 13x - 6$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 - 13x^2 + 13x - 1 = 5 - 4x^2$

Q6 **a)** Show that $(x+3)$ is a factor of $f(x) = 3x^3 + 20x^2 + 29x - 12$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $3x^3 + 22x^2 + 29x - 19 = 2x^2 - 7$

Q7 **a)** Show that $(x+1)$ is a factor of $f(x) = 2x^3 - 8x^2 + 2x + 12$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 - 11x^2 + x + 13 = 1 - x - 3x^2$

Q8 **a)** Show that $(x+3)$ is a factor of $f(x) = 2x^3 + x^2 - 12x + 9$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $2x^3 + 6x^2 - 13x + 9 = 5x^2 - x$

Q9 **a)** Show that $(x-2)$ is a factor of $f(x) = x^3 - 2x^2 - x + 2$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $x^3 - 2x^2 + 8x + 1 = 9x - 1$

Q10 **a)** Show that $(x-4)$ is a factor of $f(x) = 3x^3 - 25x^2 + 56x - 16$ and hence factorise $f(x)$ fully.

b) Hence solve the equation $3x^3 + 5x^2 + 27x - 15 = 30x^2 - 29x + 1$

RC1.1 Applying algebraic skills to solve equations

Sub-skills

- given the nature of the roots of an equation, use the discriminant to find an unknown

Q11 Each of the following functions has 2 real and distinct roots.

Calculate the range of values for k so that the function will maintain 2 real and distinct roots.

a) $f(x) = kx^2 + 6x + 8$

b) $f(x) = 3x^2 + kx + 12$

c) $f(x) = 4x^2 - 2x + k$

d) $f(x) = kx^2 + 7x + 12$

e) $f(x) = -2x^2 + kx + 2$

f) $f(x) = -4x^2 + x + k$

g) $f(x) = kx^2 - 5x + 6$

h) $f(x) = 3x^2 + kx + 8$

i) $f(x) = -x^2 + x + k$

j) $f(x) = kx^2 - 6x + 8$

Q12 For what values of p does the equation $x^2 - 2x + p = 0$ have:

a) equal roots **b)** real and distinct roots **c)** non real roots

Q13 Find m given that $x^2 + mx + x + 9 = 0$ has equal roots.

Q14 Find the values of a, b and c if each of the equations have equal roots

a) $ax^2 + 4x + 2 = 0$ **b)** $3x^2 + bx + 3 = 0$ **c)** $x^2 + 6x + c = 0$

Q15 For what value of t does $tx^2 + 6x + t = 0$ have equal roots?

Q16 Find the value of q if $x^2 + (q - 3)x + 1 = 0$ has equal roots.

Q17 Find the range of values of m for which $5x^2 - 3mx + 5 = 0$ has two real and distinct roots.

Q18 For what value of k does the graph $y = kx^2 - 3kx + 9$ touch the x axis.

Q19 Find the values for n which ensure that the following equations have equal roots:

a) $\frac{x^2+1}{x} = n$ **b)** $\frac{(x-2)^2}{x^2+2} = n$

Q20 find k if $(2k - 2)x^2 + 24x + k = 0$ has:

a) equal roots **b)** real and distinct roots

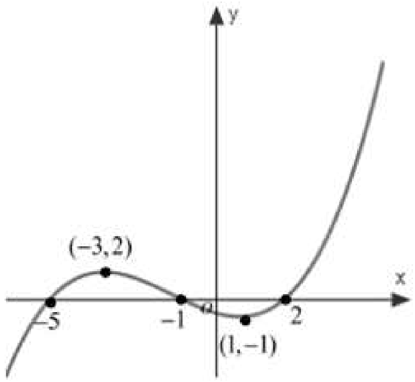
EF1.3 Applying algebraic and trigonometric skills to functions.

Sub-skills

- identifying and sketching related algebraic functions

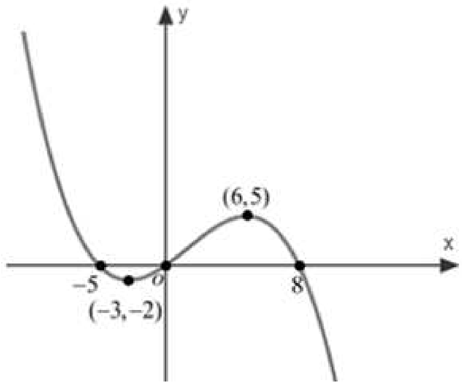
Q21 The graph of $y = f(x)$ is shown opposite.

a) Sketch $y = f(x-2)$
b) Sketch $y = f(x)+5$
c) Sketch $y = f(x-1)+2$
d) Sketch $y = f(-x)$
e) Sketch $y = -2f(x-4)$



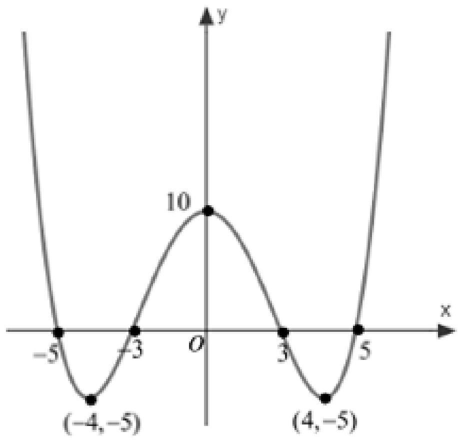
Q22 The graph of $y = g(x)$ is shown opposite.

a) Sketch $y = g(x+3)$
b) Sketch $y = g(x)-4$
c) Sketch $y = g(x-2)-1$
d) Sketch $y = g(-x)$
e) Sketch $y = 2g(x+1)$



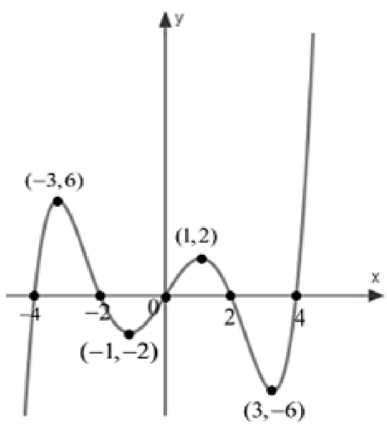
Q23 The graph of $y = h(x)$ is shown opposite.

a) Sketch $y = h(x-1)$
b) Sketch $y = h(x)+2$
c) Sketch $y = h(x+1)+4$
d) Sketch $y = -h(x)$
e) Sketch $y = 2h(x)$



Q24 The graph of $y = k(x)$ is shown opposite.

a) Sketch $y = k(x-3)$
b) Sketch $y = k(x)-2$
c) Sketch $y = k(x-1)+3$
d) Sketch $y = -k(x)$
e) Sketch $y = 4k(x-3)$



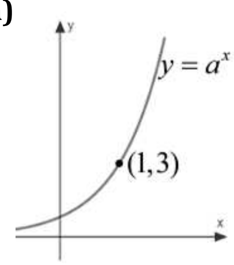
EF1.3 Applying algebraic and trigonometric skills to functions.

Sub-skills

- identifying and sketching related algebraic functions

Q25 For each of the following graphs identify the value of a.

a)



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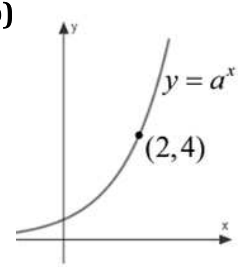

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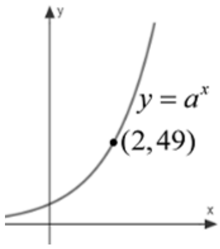

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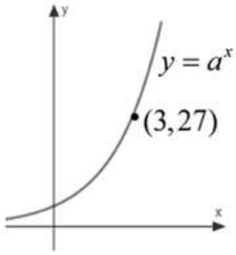

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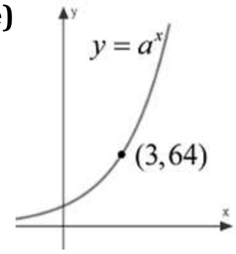

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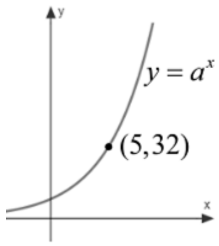

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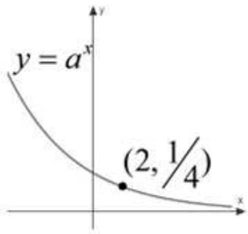

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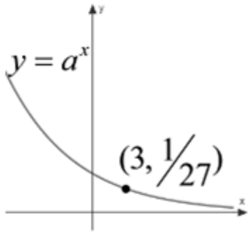

EF1.3 Applying algebraic and trigonometric skills to functions.

Sub-skills

- determining composite and inverse functions (knowledge and use of the terms domain and range is expected)

Q26 Write down the equation of the inverse functions for the following.

a)	$f(x) = 2^x$	**b)**	$f(x) = 3^x$	**c)**	$f(x) = 4^x$	**d)**	$f(x) = 7^x$
e)	$f(x) = a^x$	**f)**	$f(x) = 10^x$	**g)**	$f(x) = e^x$	**h)**	$f(x) = 5^x$
i)	$f(x) = \left(\frac{1}{2}\right)^x$	**j)**	$f(x) = 9^x$	**k)**	$f(x) = \left(\frac{2}{3}\right)^x$	**l)**	$f(x) = \left(\frac{1}{5}\right)^x$

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EF1.3 Applying algebraic and trigonometric skills to functions

Sub-skills

- determining composite and inverse functions

  (knowledge and use of the terms domain and range is expected)

Q27 Determine the inverse function for each of the following, assuming an appropriate domain and range.

a) $f(x) = 5x - 2$ **b)** $f(x) = 3 - 4x$ **c)** $f(x) = 2x + 7$

d) $f(x) = \frac{x-2}{4}$ **e)** $f(x) = \frac{3}{2} - 5x$ **f)** $f(x) = 2(3x - 1)$

g) $f(x) = 5x^2$ **h)** $f(x) = 3(x-2)^2$ **i)** $f(x) = 2(3x+1)^2$

EF1.3 Applying algebraic and trigonometric skills to functions

EF#2.2 Explaining a solution and, where appropriate, relating it to context

Sub-skills

- determining composite and inverse functions

  (knowledge and use of the terms domain and range is expected)

Q28 In each of the examples below, functions f and g are defined on suitable domains.

For each question, obtain expressions for $f(g(x))$ and $g(f(x))$ in their simplest form.

a) $f(x) = x - 2$ $g(x) = x^2$

b) $f(x) = 2x + 1$ $g(x) = x^2 - 2x + 1$

c) $f(x) = 5x - 3$ $g(x) = \frac{x+3}{5}$, explain the relationship between $f(x) and\ g(x)$.

d) $f(x) = 4 - 3x$ $g(x) = \frac{4-x}{3}$, explain the relationship between $f(x) and\ g(x)$.

e) $f(x) = x^2 + 1$ $g(x) = (x+1)^2$

f) $f(x) = \frac{2}{3}x - 4$ $g(x) = \frac{3}{2}x + 6$, explain the relationship between $f(x) and\ g(x)$.

g) $f(x) = 3 - 2x^2$ $g(x) = x - 1$

h) $f(x) = 3x + 5$ $g(x) = \frac{2}{x+1}$

i) $f(x) = \frac{x-2}{x+1}$ $g(x) = \frac{1}{x-2}$

j) $f(x) = \sqrt{x+4}$ $g(x) = x^2 - 4$, explain the relationship between $f(x) and\ g(x)$.

EF1.3 Applying algebraic and trigonometric skills to functions

Sub-skills

- identifying and sketching related trigonometric functions

Q29 Sketch each of the following trigonometric functions, where $0 \leq x \leq 2\pi$

a)	$y = \sin x$	**b)**	$y = \cos x$
c)	$y = 2\sin x$	**d)**	$y = 3\cos x$
e)	$y = \sin x + 3$	**f)**	$y = \cos x - 4$
g)	$y = \sin 2x$	**h)**	$y = \cos\left(\frac{1}{2}x\right)$
i)	$y = -\sin x$	**j)**	$y = -\cos x$
k)	$y = 2\sin 3x - 1$	**l)**	$y = 2 - \cos 3x$

EF1.3 Applying algebraic and trigonometric skills to functions

EF#2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy

Sub-skills

- identifying and sketching related trigonometric functions

Q30 Use the information provided in the table below to identify the equation of the curves.

All equations are of the form $y = a\sin bx + c$ *or* $y = a\cos bx + c$.

	Type of curve	Domain	Maximum occurs at	Minimum occurs at
a)	sine	$0 \leq x \leq \pi$	$\left(\frac{\pi}{4}, 1\right)$	$\left(\frac{3\pi}{4}, -1\right)$
b)	cosine	$0 \leq x \leq \frac{\pi}{2}$	$(0, 1)\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{\pi}{4}, -1\right)$
c)	sine	$0 \leq x \leq 2\pi$	$\left(\frac{\pi}{2}, 3\right)$	$\left(\frac{3\pi}{2}, 1\right)$
d)	cosine	$0 \leq x \leq \pi$	$(0, 3)\ (\pi, 3)$	$\left(\frac{\pi}{2}, -3\right)$
e)	sine	$0 \leq x \leq 2\pi$	$\left(\frac{3\pi}{2}, 5\right)$	$\left(\frac{\pi}{2}, 3\right)$
f)	cosine	$0 \leq x \leq 2\pi$	$(\pi, -1)$	$(0, -3)\ (2\pi, -3)$
g)	sine	$0 \leq x \leq \pi$	$\left(\frac{\pi}{4}, 5\right)$	$\left(\frac{3\pi}{4}, 1\right)$
h)	cosine	$0 \leq x \leq \pi$	$(0, 2)\ (\pi, 2)$	$\left(\frac{\pi}{2}, -4\right)$
i)	sine	$0 \leq x \leq \frac{2\pi}{3}$	$\left(\frac{\pi}{6}, 4\right)$	$\left(\frac{\pi}{2}, -2\right)$
j)	cosine	$0 \leq x \leq \frac{\pi}{2}$	$\left(\frac{\pi}{4}, 1\right)$	$(0, -3)\left(\frac{\pi}{2}, -3\right)$

Q31 Use your answers from Q30 to sketch the graphs of the trigonometric functions.

ANSWERS

Q1	**a)**	$f(x) = (x+1)(x-1)(2x-5)$	**b)**	$x = -1,\ 1,\ \frac{5}{2}$
Q2	**a)**	$f(x) = (x-3)(x-1)(3x-5)$	**b)**	$x = 3,\ 1,\ \frac{5}{3}$
Q3	**a)**	$f(x) = (x-1)(x-2)(2x+3)$	**b)**	$x = 1,\ 2,\ -\frac{3}{2}$
Q4	**a)**	$f(x) = (x+3)(x+5)(x-4)$	**b)**	$x = -3,\ -5,\ 4$
Q5	**a)**	$f(x) = (x-2)(x-1)(2x-3)$	**b)**	$x = 2,\ 1,\ \frac{3}{2}$
Q6	**a)**	$f(x) = (x+3)(x+4)(3x-1)$	**b)**	$x = -3,\ -4,\ \frac{1}{3}$
Q7	**a)**	$f(x) = 2(x+1)(x-3)(x-2)$	**b)**	$x = -1,\ 3,\ 2$
Q8	**a)**	$f(x) = (x+3)(x-1)(2x-3)$	**b)**	$x = -3,\ 1,\ \frac{3}{2}$
Q9	**a)**	$f(x) = (x-2)(x-1)(x+1)$	**b)**	$x = -1,\ 1,\ 2$
Q10	**a)**	$f(x) = (x-4)(x-4)(3x-1)$	**b)**	$x = 4,\ \frac{1}{3}$
Q11	**a)**	$k < \frac{9}{8}$	**b)**	$k > 12,\ k < -12$
	c)	$k < \frac{1}{4}$	**d)**	$k < \frac{49}{48}$
	e)	$All\ values\ of\ k$	**f)**	$k > -\frac{1}{16}$
	g)	$k < \frac{25}{24}$	**h)**	$k > 4\sqrt{6},\quad k < -4\sqrt{6}$
	i)	$k > -\frac{1}{4}$	**j)**	$k < \frac{9}{8}$

Q12 **a)** $p = 1$ **b)** $p \leq 1$ **c)** $p > 1$

Q13 $m = 5, -7$

Q14 **a)** $a = 2$ **b)** $b = \pm 6$ **c)** $c = 9$

Q15 $t = \pm 3$

Q16 $q = 1,\ 5$

Q17 $m \geq \frac{10}{3},\ m \leq -\frac{10}{3}$

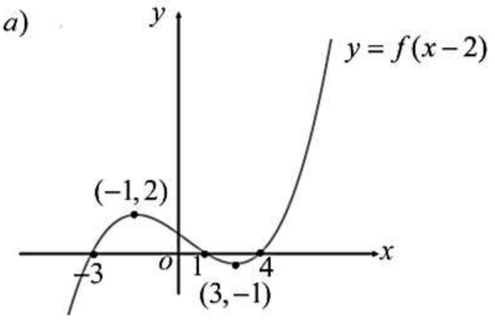
Q18 $k = 4,\quad k \neq 0\ as\ then\ y = 9.\ \ This\ line\ cannot\ touch\ the\ x\ axis$

Q19 **a)** $n = \pm 2$ **b)** $n = 0,\quad 3$

Q20 **a)** $k = 9, -8$ **b)** $k < -8,\ \ k > 9$

Q21

a)



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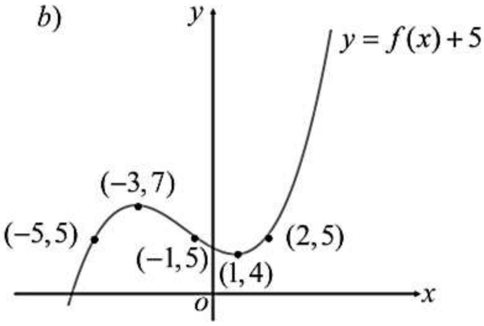

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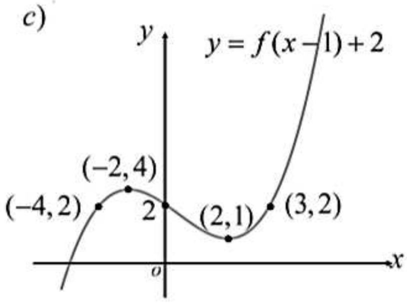

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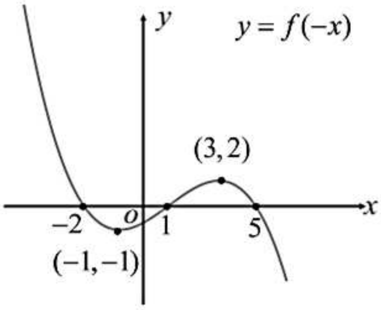

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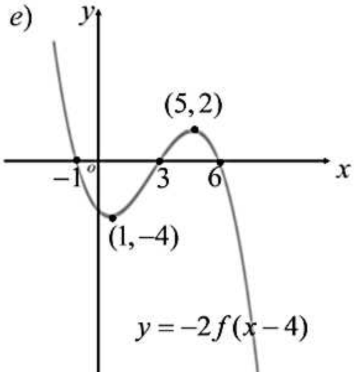
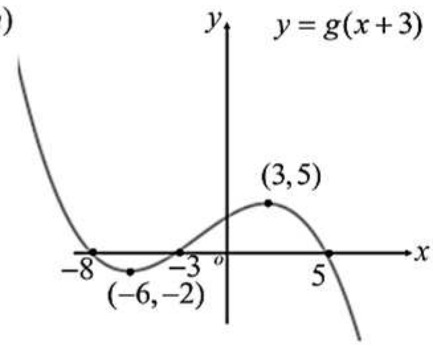


Q22

a)



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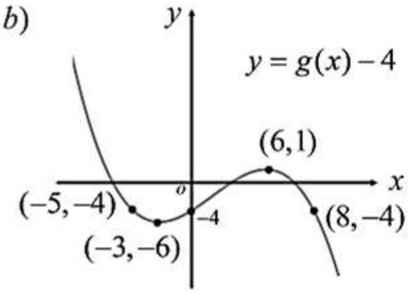

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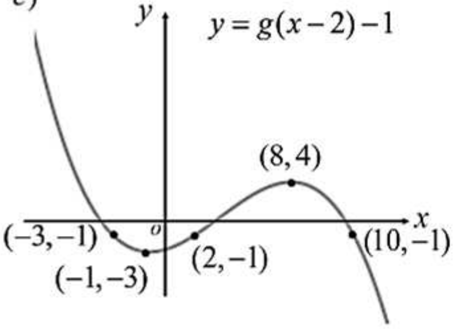

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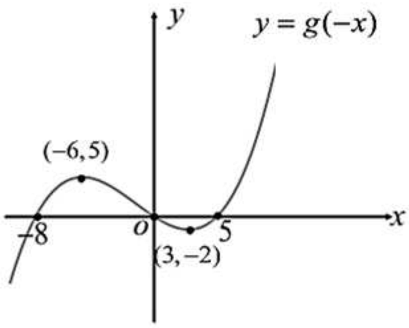

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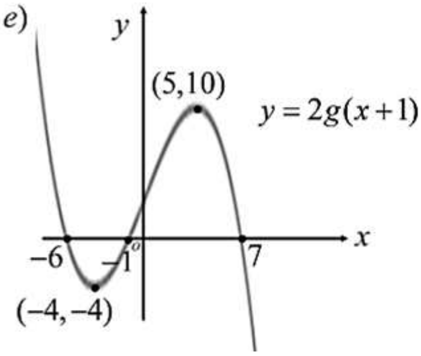
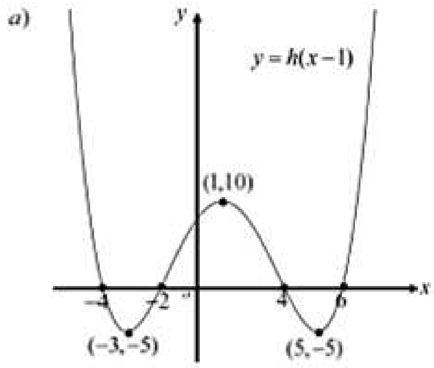
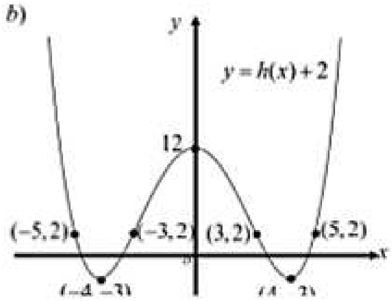
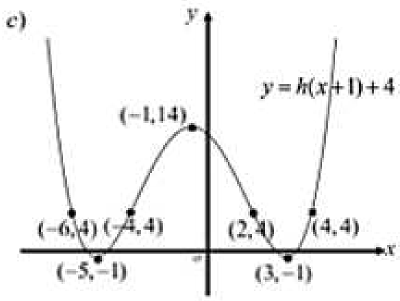
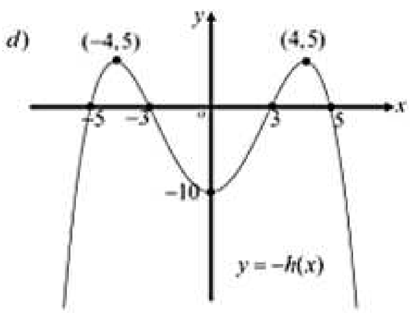


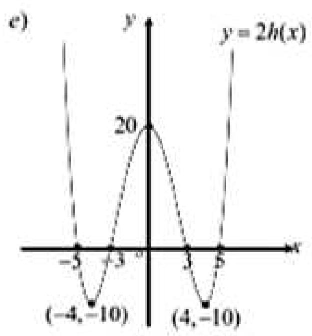
Q23


a)
y = h(x − 1)
(1,10)
−4
−2
4
6
(−3, −5)
(5, −5)

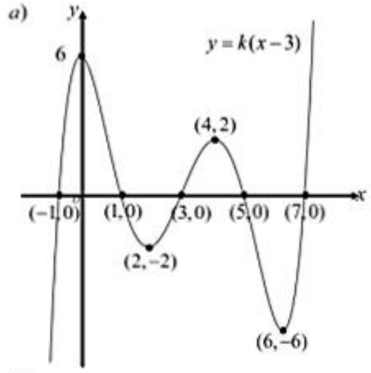

b)
y = h(x) + 2
12
(−5, 2)
(−3, 2)
(3, 2)
(5, 2)

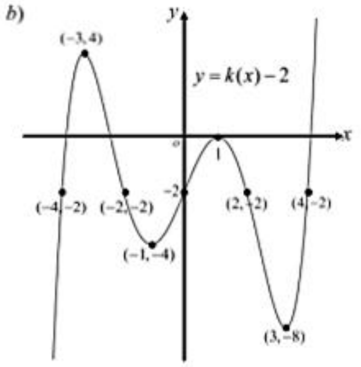

c)
y = h(x + 1) + 4
(−1,14)
(−6, 4)
(−4, 4)
(2, 4)
(4, 4)
(−5, −1)
(3, −1)

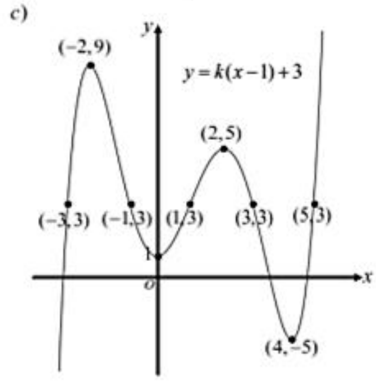

d)
(−4, 5)
(4, 5)
−5
−3
3
5
−10
y = −h(x)

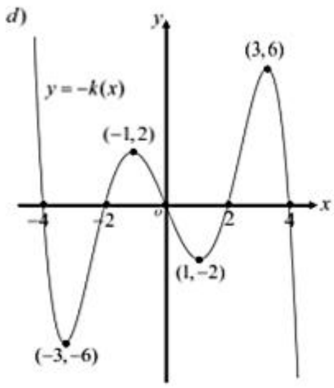

e)
y = 2h(x)
20
−5
−3
3
5
(−4, −10)
(4, −10)

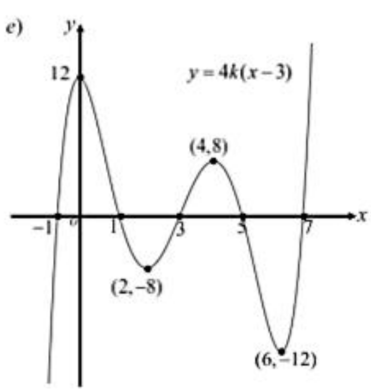
Q24


a)
y = k(x − 3)
6
(4, 2)
(−1, 0)
(1, 0)
(3, 0)
(5, 0)
(7, 0)
(2, −2)
(6, −6)


b)
(−3, 4)
y = k(x) − 2
1
−2
(−4, −2)
(−2, −2)
(2, −2)
(4, −2)
(−1, −4)
(3, −8)


c)
(−2, 9)
y = k(x − 1) + 3
(2, 5)
(−3, 3)
(−1, 3)
(1, 3)
(3, 3)
(5, 3)
1
(4, −5)


d)
(3, 6)
y = −k(x)
(−1, 2)
−4
−2
2
4
(1, −2)
(−3, −6)


e)
12
y = 4k(x − 3)
(4, 8)
−1
1
3
5
7
(2, −8)
(6, −12)

Q25

a) $a = 3$ **b)** $a = 2$

c) $a = 7$ **d)** $ka = 3$

e) $a = 4$ **f)** $a = 2$

g) $a = \frac{1}{2}$ **h)** $a = \frac{1}{3}$

Q26

a) $\boldsymbol{y} = \log_2 x$ **b)** $\boldsymbol{y} = \log_3 x$

c) $\boldsymbol{y} = \log_4 x$ **d)** $\boldsymbol{y} = \log_7 x$

e) $\boldsymbol{y} = \log_a x$ **f)** $\boldsymbol{y} = \log_{10} x$

g) $\boldsymbol{y} = \log_e x \;\; or \;\ln x$ **h)** $\boldsymbol{y} = \log_5 x$

i) $\boldsymbol{y} = \log_{\frac{1}{2}} x$ **j)** $\boldsymbol{y} = \log_9 x$

k) $\boldsymbol{y} = \log_{\frac{2}{3}} x$ **l)** $\boldsymbol{y} = \log_{\frac{1}{5}} x$

Q27

a) $f^{-1}(x) = \frac{x+2}{5}$ **b)** $f^{-1}(x) = \frac{3-x}{4}$ **c)** $f^{-1}(x) = \frac{x-7}{2}$

d) $f^{-1}(x) = 4x + 2$ **e)** $f^{-1}(x) = \frac{3-2x}{10}$ **f)** $f^{-1}(x) = \frac{x+2}{6}$

g) $f^{-1}(x) = \sqrt{\frac{x}{5}}$ **h)** $f^{-1}(x) = \sqrt{\frac{x}{3}} + 2$ **i)** $f^{-1}(x) = \frac{\sqrt{2}(\sqrt{x}-2)}{6}$

Q28

a) $f\big(g(x)\big) = x^2 - 2$ $g\big(f(x)\big) = (x-2)^2$

b) $f\big(g(x)\big) = 2x^2 - 4x + 3$ $g\big(f(x)\big) = 4x^2$

c) $f\big(g(x)\big) = x$ $g\big(f(x)\big) = x$

$f(x) = g^{-1}(x) \;\; and \; f^{-1}(x) = g(x) \; (inverse \; of \; each \; other)$

d) $f\big(g(x)\big) = x$ $g\big(f(x)\big) = x$

$f(x) = g^{-1}(x) \;\; and \; f^{-1}(x) = g(x) \; (inverse \; of \; each \; other)$

e) $f\big(g(x)\big) = (x+1)^4 + 1$ $g\big(f(x)\big) = (x^2+2)^2$

f) $f\big(g(x)\big) = x$ $g\big(f(x)\big) = x$

$f(x) = g^{-1}(x) \;\; and \; f^{-1}(x) = g(x) \; (inverse \; of \; each \; other)$

g) $f\big(g(x)\big) = 1 + 4x - 2x^2$ $g\big(f(x)\big) = 2(1+x)(1-x)$

h) $f\big(g(x)\big) = \frac{6}{x+1} + 5$ $g\big(f(x)\big) = \frac{2}{3(x+2)}$

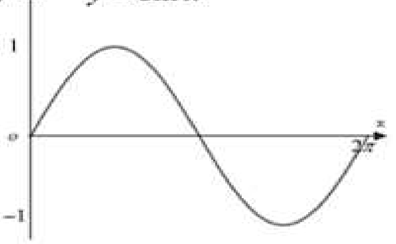
i) $f\big(g(x)\big) = \frac{5-2x}{x-1}$ $g\big(f(x)\big) = -\frac{(x+1)}{(x+4)}$

j) $f\big(g(x)\big) = x$ $g\big(f(x)\big) = x$

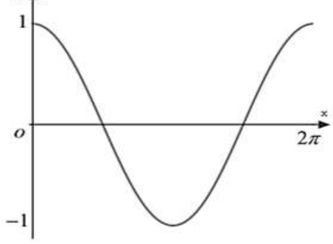
$f(x) = g^{-1}(x) \;\; and \; f^{-1}(x) = g(x) \; (inverse \; of \; each \; other)$

Q29

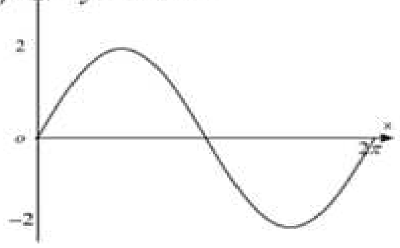
a) $y = \sin x$



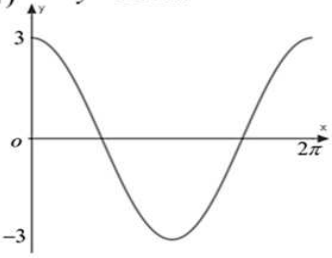
b) $y = \cos x$



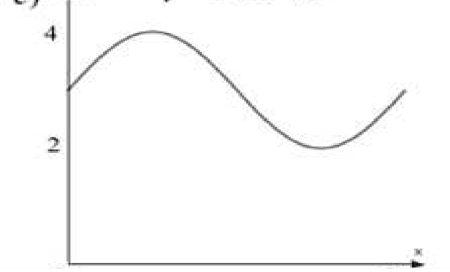
c) $y = 2\sin x$



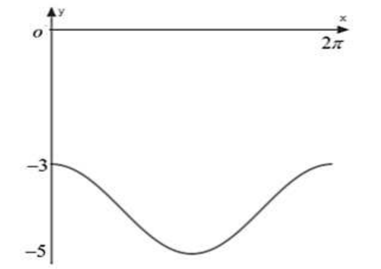
d) $y = 3\cos x$



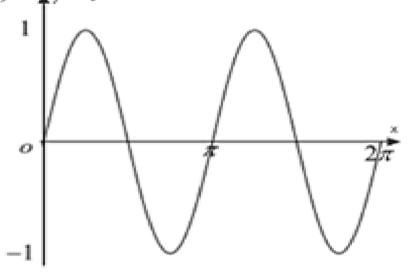
e) $y = \sin x + 3$



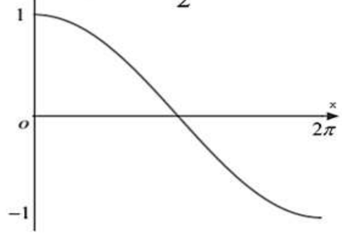
f) $y = \cos x - 4$



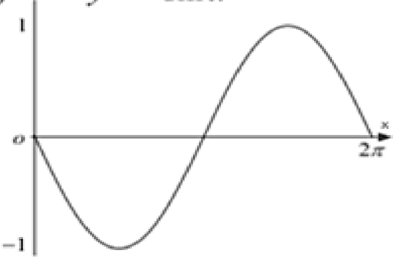
g) $y = \sin 2x$



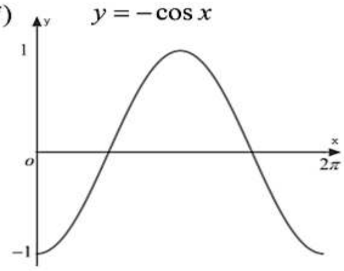
h) $y = \cos \frac{1}{2} x$



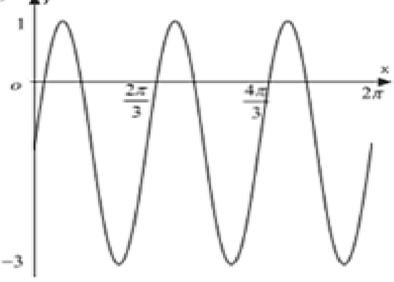
i) $y = -\sin x$



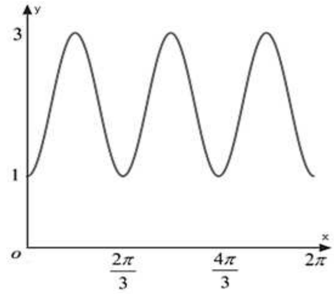
j) $y = -\cos x$



k) $y = 2\sin 3x - 1$



l) $y = 2 - \cos 3x$



Q30

a)	$y = sin\ 2x$	**b)**	$y = \cos 4x$
c)	$y = sin\ x + 2$	**d)**	$y = 3\cos 2x$
e)	$y = -sin\ x + 4$	**f)**	$y = -\cos x - 2$
g)	$y = 2sin\ 2x + 3$	**h)**	$y = 3\cos 2x - 1$
i)	$y = 3sin\ 3x + 1$	**j)**	$y = -2\cos 4x - 1$

Q31