Mathematics Higher

Revision Materials

Differentiation & Circle Skills Builder

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

Unit	Assessment standard	Sub-skills
H4LD 76 Relationships and Calculus	RC1.3 Applying calculus skills of differentiation	The sub-skills in the Assessment Standard are: ♦ differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x* ♦ differentiating $k\sin x,\ k\cos x$ ♦ determining the equation of a tangent to a curve at a given point by differentiation
	RC#2.2 Explaining a solution and, where appropriate, relating it to context	Assessment Standard 2.2 is transferable across Units. For candidates undertaking the Course, Assessment Standard 2.2 should be achieved on **at least two** occasions from across the Course.
H22J 76 Applications	AP1.4 Applying calculus skills to optimisation and area	The sub-skills in the Assessment Standard are: ♦ determining the optimal solution for a given problem
	AP1.2 Applying algebraic skills to circles	The sub-skills in the Assessment Standard are: ♦ determining and using the equation of a circle ♦ using properties of tangency in the solution of a problem
	AP#2.2 Explaining a solution and, where appropriate, relating it to context	Assessment Standard 2.2 is transferable across Units. For candidates undertaking the Course, Assessment Standard 2.2 should be achieved on **at least two** occasions from across the Course.

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x***

Q1 Differentiate the following expressions

a) x^2 **b)** x^4 **c)** x^7 **d)** $3x^5$

e) $\frac{1}{x}$ **f)** $\frac{2}{x^3}$ **g)** $\frac{-3}{x^5}$ **h)** $\frac{x}{6x^4}$

i) $x^{\frac{1}{2}}$ **j)** $x^{\frac{1}{3}}$ **k)** $x^{\frac{2}{3}}$ **l)** $3x^{\frac{5}{2}}$

m) $x^2 - 2x$ **n)** $x^3 + 2x^2 - 1$ **o)** $x^7 + 5x - 3$ **p)** $6x^5 - 3x^2 + 7$

q) $x^2 + 4x - 1 + \frac{1}{x^3}$ **r)** $5x^4 - \frac{3}{2x^2} - \frac{1}{x^3}$ **s)** $x^2 + 1 + \frac{1}{\sqrt{x}}$

t) $(x+3)^2$ **u)** $(2x+7)(3x-1)$ **v)** $(x+1)(x-2)(2x+1)$

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **differentiating** $k\sin x,\ k\cos x$

Q2 Differentiate the following trig functions

a) $f(x) = \sin x$ **b)** $f(x) = \cos x$ **c)** $f(x) = 2\cos x$

d) $f(x) = \frac{1}{2}\sin x$ **e)** $f(x) = 4\cos x$ **f)** $f(x) = -3\sin x$

g) $f(x) = -5\cos x$ **h)** $f(t) = 2\sin t$ **i)** $g(r) = -\sqrt{2}\pi\cos r$

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x***
- **differentiating** $k\sin x,\ k\cos x$

Q3 Differentiate the following equations

a) $y = \frac{1}{x^3} - \frac{x^{\frac{2}{3}}}{4}$ **b)** $y = \cos x + 3x^2$ **c)** $y = \sqrt[3]{x} + \frac{3}{\sqrt[3]{x}}$

d) $y = \frac{1}{2}\sin x + \sqrt[3]{x^4}$ **e)** $y = 7x^{\frac{5}{3}} - \frac{2}{x^6}$ **f)** $y = x(1 + \sqrt{x})$

g) $y = \frac{1+5x^2}{x}$ **h)** $y = \frac{7x^2 - 2x\sin x}{x}$ **i)** $y = \frac{x^2 - \sqrt{x}\,cosx}{x^{\frac{1}{2}}}$

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **Differentiating a composite function using the chain rule**
  (Extension-Exam Level)

Q4 Use the Chain Rule to help differentiate the following

a) $y = (2x^2 - 3)^4$ **b)** $y = \sin(3x)$ **c)** $f(x) = \frac{6}{(x^3 - 2x)}$

c) $y = (x^2 - 3x^4)^{\frac{1}{2}}$ **d)** $y = (\cos x)^2$ **e)** $f(x) = \frac{-1}{(x^2+3x)^3}$

f) $y = (3x^2 - 2x + 7)^9$ **g)** $y = (\sin(3x))^2$ **h)** $f(x) = \frac{3}{\sqrt{(x^2 - x)}}$

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **determining the equation of a tangent to a curve at a given point by differentiation**
- **differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x***

Q5 Find the gradient and the equation of the tangent to the following:

a) $y = 3x^2 \ \ at\ x = 2$ **b)** $y = x^2 + 2x\ at\ x = 1$ **c)** $y = x^4 \ \ at\ x = 1$

d) $y = \sqrt{x} \ \ at\ x = 4$ **e)** $y = x^{\frac{3}{2}} \ \ at\ x = 1$ **f)** $y = \frac{1}{x^2} \ \ at\ x = -1$

g) $y = 16 - 3x^2 \ \ at\ x = -2$ **h)** $y = (x-1)^2 \ \ at\ x = 1$

Q6 **a)** Find the point on the parabola $y = x^2 - 4x + 1$ which has a gradient of 2.

b) Find the equation of the tangent.

Q7 **a)** Find the points on the curve $y = \frac{1}{3}x^3$ at which the tangents have a gradient of 9.

b) Find the equations of the tangents.

Q8 The point $A(1,2)$ lies on the curve $y = x^3 + x^2$.

a) Find the equation of the tangent at A.

b) Find the coordinates where the tangent meets the curve again.

Q9 **a)** Find the equations of the tangents to $y = 2x^2 \ \ at\ x = 1\ and\ x = -1$.

b) Find the point of intersection of the tangents.

Q10 The curve $y = (x-1)(x^2+7)$ meets the $x - axis$ at P and the $y - axis$ at Q.

Find the equations of the tangents at $P\ and\ Q$.

Q11 **a)** Find the equation of the tangent to the curve $y = x^3 - 2x^2 + 7 \;\; at\; x = 2$.

b) Find the coordinates of the point where the tangent cuts the curve again.

Q12 **a)** Find the points of contact on the curve $y = x^3 - 4x$ for tangents with a gradient of 8.

b) Find the equations of these tangents.

Q13 **a)** Show there is only one tangent to the curve $y = 3x^2 + 5x$ with gradient of 11.

b) Find the equation of this tangent.

RC1.3 Applying calculus skills of differentiation .

Sub-skills

- **determining the equation of a tangent to a curve at a given point by differentiation**
- **differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x* (Extension Exam Level)**

Q14 Find the stationary points of the following curves, stating and justifying their nature.

a) $y = x^2 + 1$ **b)** $y = 3 - 2x^2$ **c)** $y = x^3$

d) $y = 3x - x^3$ **e)** $y = 3x^4 - 4x^3$ **f)** $y = x^4 - 2x^2 + 5$

g) $y = x^3 + 3x$ **h)** $y = x^2 - 4x$ **i)** $y = (1 - 2x)(1 + 2x)$

j) $y = (2 - x)^2$ **k)** $y = 5 + 4x - x^2$ **l)** $y = 3x^5 - 5x^3 + 2$

RC1.3 Applying calculus skills of differentiation .

AP1.4 Applying calculus skills to optimisation and area

Sub-skills

- **differentiating an algebraic function which is, or can be simplified to, an expression in powers of *x***
- **determining the optimal solution for a given problem**

Q15 The sum of two numbers x *and* y is 12. ie $x + y = 12$.

Their Product P is given by $P(x) = x(12 - x)$

Find the values of x *and* y which maximise P.

Justify your answer.

Q16 The difference of two numbers x *and* y is 50. ie $y - x = 50$

Their Product P is given by $P(x) = x(x + 50)$

Find the values of x *and* y which minimise P.

Justify your answer.

Q17 The product of two positive integers x *and* y *is* 36. ie $xy = 36$

Their sum is given by $S(x) = x + \frac{36}{x}$

Find the values of x *and* y which minimise S.

Justify your answer.

Q18 A rectangle with sides x and y has a perimeter of 60 cm. $2x + 2y = 60$.

Its area is given by $A(x) = x(30 - x)$

Find the values of x *and* y which maximise A.

Justify your answer.

Q19 A rectangle has lengths x *and* y and an area of $100cm^2$. $xy = 100$

The perimeter is given by $P(x) = 2x + \frac{200}{x}$

Find the values of x *and* y which minimise P.

Justify your answer.

Q20 A cuboid is open at the top and has a square base of $x\ cm$, a height of $y\ cm$ and a volume of 13.5 cm^3.

The surface area of the cuboid is given by $S(x) = x^2 + \frac{54}{x}$.

Find the dimensions which give the minimum surface area.

Justify your answer.

Q21 The area of a triangle enclosed in a rectangle if given by $A(x) = 50 - 10x + x^2$.

Find the value of x which minimises the area of the triangle.

Justify your answer.

Q22 A parallelogram is trapped within a rectangle.

The area of the parallelogram is given by $A(x) = 96 - 20x + 2x^2$.

Find the minimum area of the parallelogram.

Justify your answer.

CIRCLES

AP1.2 Applying algebraic skills to circles

Sub-skills

- **determining and using the equation of a circle**

Q23 Write down the equation of the circles with these centres and radii.

a) $Centre\ (1,2)\ \ radius\ 4$ **b)** $Centre\ (-2,-1)\ \ radius\ 5$

c) $Centre\ (-1,0)\ \ radius\ 3$ **d)** $Centre\ (2,-4)\ \ radius\ 13$

e) $Centre\ (0,2)\ \ radius\ 2$ **f)** $Centre\ (-3,4)\ \ radius\ 7$

g) $Centre\ (2,-7)\ \ radius\ 6$ **h)** $Centre\ (2,-5)\ \ radius\ 11$

i) $Centre\ (5,-13)\ \ radius\ 4$ **j)** $Centre\ (-1,\ 4)\ \ radius\ 2$

k) $Centre\ (6,-6)\ \ radius\ 8$ **l)** $Centre\ (-3,1)\ \ radius\ 5$

Q24 Identify the centre and the radii of the following circles

a) $(x-1)^2+(y-3)^2=16$ **b)** $(x-11)^2+(y+2)^2=36$

c) $(x-2)^2+(y+4)^2=25$ **d)** $(x+7)^2+(y-3)^2=64$

e) $(x+1)^2+(y-3)^2=128$ **f)** $(x-6)^2+(y+1)^2=96$

g) $(x+5)^2+(y+7)^2=49$ **h)** $(x-3)^2+(y-4)^2=17$

i) $(x+2)^2+(y-9)^2=81$ **j)** $(x-13)^2+(y+11)^2=104$

k) $(x-1)^2+(y+6)^2=24$ **l)** $(x-8)^2+(y+1)^2=54$

Q25 Find the equation for each of these circles with Centre C, and passing through Point P.

a) $C(0,4)$, $P(3,1)$ **b)** $C(-1,2)$, $P(5,-3)$ **c)** $C(-7,4)$, $P(2,-1)$

d) $C(-3,1)$, $P(8,2)$ **e)** $C(3,2)$, $P(4,4)$ **f)** $C(2,-9)$, $P(1,-7)$

g) $C(3,1)$, $P(5,-2)$ **h)** $C(2,-3)$, $P(7,-2)$ **i)** $C(-2,-3)$, $P(4,0)$

j) $C(5,0)$, $P(1,3)$ **k)** $C(3,-5)$, $P(-2,-7)$ **l)** $C(8,1)$, $P(-3-7)$

Q26 Identify the centre and radii of the following circles

a) $x^2+y^2-4x-8y+16=0$ **b)** $x^2+y^2-8x-2y+13=0$

c) $x^2+y^2-2x+2y-5=0$ **d)** $x^2+y^2+4x+6y+3=0$

e) $x^2+y^2+2x+2y-7=0$ **f)** $x^2+y^2+6x-8y+24=0$

g) $x^2+y^2+2x-2y-2=0$ **h)** $x^2+y^2-6x-2y+1=0$

i) $x^2+y^2-10x+8y+32=0$ **j)** $x^2+y^2-8x-6y+9=0$

AP1.2 Applying algebraic skills to circles

AP#2.2 Explaining a solution and, where appropriate, relating it to context

Sub-skills

- **determining and using the equation of a circle**

Q27 For each pair of equations, state if the circles intersect.

Justify your answer.

a) $(x-1)^2+(y-2)^2=20$ and $x^2+y^2-6x+8y+12=0$

b) $(x-2)^2+(y-5)^2=17$ and $x^2+y^2+8x+2y+7=0$

c) $(x-7)^2+(y+3)^2=24$ and $x^2+y^2+8x-6y+11=0$

d) $(x+2)^2+(y-1)^2=14$ and $x^2+y^2-8x+6y-11=0$

e) $(x+1)^2+(y+3)^2=21$ and $x^2+y^2+6x-4y+7=0$

f) $(x-12)^2+(y+10)^2=52$ and $x^2+y^2+16x-4y-182=0$

g) $(x+4)^2+(y+2)^2=28$ and $x^2+y^2+6x-10y-16=0$

h) $(x+6)^2+(y-5)^2=57$ and $x^2+y^2+8x-6y+5=0$

AP1.2 Applying algebraic skills to circles

Sub-skills

- **using properties of tangency in the solution of a problem**

Q28 Find the equation of the tangent at Point P for each of these circles with Centre C.

a) $C(2,3)$, $P(0,4)$ **b)** $C(-4,6)$, $P(0,-4)$ **c)** $C(-2,-3)$, $P(-6,8)$

d) $C(-10,3)$, $P(-8,-1)$ **e)** $C(-8,-10)$, $P(-3,-6)$ **f)** $C(-1,8)$, $P(1,-7)$

g) $C(4,6)$, $P(-3,5)$ **h)** $C(-1,-2)$, $P(-5,-7)$ **i)** $C(-5,-7)$, $P(2,-8)$

j) $C(-6,-4)$, $P(1,0)$ **k)** $C(0,6)$, $P(-3,-7)$ **l)** $C(4,1)$, $P(9,-10)$

Answers

Q1

a) $2x$ **b)** $4x^3$ **c)** $7x^6$ **d)** $15x^4$

e) $-\frac{1}{x^2}$ **f)** $-\frac{6}{x^4}$ **g)** $\frac{15}{x^6}$ **h)** $-\frac{1}{2x^4}$

i) $\frac{1}{2}x^{-\frac{1}{2}}$ **j)** $\frac{1}{3}x^{-\frac{2}{3}}$ **k)** $\frac{2}{3}x^{-\frac{1}{3}}$ **l)** $\frac{15}{2}x^{\frac{3}{2}}$

m) $2x-2$ **n)** $3x^2+4x$ **o)** $7x^6+5x$ **p)** $30x^4-6x$

q) $2x+4-\frac{3}{x^4}$ **r)** $20x^3+\frac{3}{x^3}+\frac{3}{x^4}$ **s)** $2x-\frac{1}{2}x^{-\frac{3}{2}}$

t) $2x+6$ **u)** $12x+19$ **v)** $6x^2-2x-5$

Q2

a) $f'(x)=\cos x$ **b)** $f'(x)=-\sin x$ **c)** $f'(x)=-2\sin x$

d) $f'(x)=\frac{1}{2}\cos x$ **e)** $f'(x)=-4\sin x$ **f)** $f'(x)=-3\cos x$

g) $f'(x)=5\sin x$ **h)** $f'(t)=2\cos t$ **i)** $g'(r)=\sqrt{2}\pi\sin r$

Q3

a) $\frac{dy}{dx}=-\frac{3}{x^4}-\frac{1}{6}x^{-\frac{2}{3}}$ **b)** $\frac{dy}{dx}=-\sin x+6x$ **c)** $\frac{dy}{dx}=\frac{1}{3}x^{-\frac{2}{3}}-x^{-\frac{4}{3}}$

d) $\frac{dy}{dx}=\frac{1}{2}\cos x+\frac{4}{3}x^{\frac{1}{3}}$ **e)** $\frac{dy}{dx}=\frac{35}{3}x^{\frac{2}{3}}+12x^{-7}$ **f)** $\frac{dy}{dx}=1+\frac{3}{2}x^{\frac{1}{2}}$

g) $\frac{dy}{dx}=-x^{-2}+5$ **h)** $\frac{dy}{dx}=7x-2\cos x$ **i)** $\frac{dy}{dx}=\frac{3}{x}x^{\frac{1}{2}}+\sin x$

Q4

a) $\frac{dy}{dx}=16x(2x^2-3)^3$ **b)** $\frac{dy}{dx}=3\cos(3x)$ **c)** $f'(x)=-6(3x^2-2)(x^3-2x)^{-2}$

c) $\frac{dy}{dx}=\frac{1}{2}(2x-12x^3)(x^2-3x^4)^{-\frac{1}{2}}$ **d)** $\frac{dy}{dx}=-2\sin x\cos x$

e) $f'(x)=-3(2x+3)(x^2+3x)^{-4}$ **f)** $\frac{dy}{dx}=9(6x-2)(3x^2-2x+7)^8$

g) $\frac{dy}{dx}=6\sin 3x\cos 3x$ **h)** $f'(x)=-\frac{3}{2}(2x-1)(x^2-x)^{-\frac{3}{2}}$

Q5

a) $m=12$, $y=12x-12$ **b)** $m=4$, $y=4x-1$

c) $m=4$, $y=4x-3$ **d)** $m=\frac{1}{4}$, $4y=x+4$

e) $m=\frac{3}{2}$, $2y=3x-1$ **f)** $m=2$, $y=2x+3$

g) $m=12$, $y=12x+28$ **h)** $m=0$, $y=0$ $(x\ axis)$

Q6 **a)** $(3,-2)$ **b)** $y=3x-11$

Q7 **a)** $(3,9)\ and\ \ (-3,-9)$ **b)** $y=9x-18\ \ and\ \ y=9x+18$

Q8 **a)** $y=5x-3$ **b)** $(-3,-18)$

Q9 **a)** $y=4x-2\ \ and\ y=-4x-2$ **b)** $(0,-2)$

Q10 $P,\ \ y=8x-8$ $Q,\ \ y=7x-7$

Q11 **a)** $y=4x-1$ **b)** $(-2,-9)$

Q12 **a)** $(2,0)\ and\ \ (-2,0)$ **b)** $y=8(x-2)\ \ and\ \ y=8(x+2)$

Q13 **a)** $Proof: Show\ as\ required$ **b)** $y=11x-3$

Q14 **All answers must be accompanied by a nature table or second derivative for justification.**

a) $\min(0,1)$ **b)** $\max(0,3)$ **c)** $(0,0) rising\ point\ of\ inflection$

d) $\max(1,2)$ $\min(-1,-2)$ **e)** $(0,0) falling\ point\ of\ inflection,\ \min(1,-1)$

f) $\min(1,4)\,,\min(-1,4)\,,\max(0,5)$ **g)** $no\ stationary\ points$

h) $\min(2,-4)$ **i)** $\max(0,1)$ **j)** $\min(2,0)$

k) $\max(2,9)$ **l)** $(0,2) falling\ point\ of\ inflection, \min(1,0)\,,\max(-1,4)$

Q15 $x=6, y=6$ plus justification

Q16 $x=-25, y=25$ plus justification

Q17 $x=6, y=6$ plus justification

Q18 $x = 15, y = 15$ plus justification

Q19 $x = 10cm, y = 10cm$ plus justification

Q20 $x = 3cm, y = 1.5cm$ plus justification

Q21 $x = 5$ plus justification

Q22 $x = 5$ plus justification

Q23

a) $(x-1)^2 + (y-2)^2 = 16$ **b)** $(x+2)^2 + (y+1)^2 = 25$

c) $(x+1)^2 + y^2 = 9$ **d)** $(x-2)^2 + (y+4)^2 = 169$

e) $x^2 + (y-2)^2 = 4$ **f)** $(x+3)^2 + (y-4)^2 = 49$

g) $(x-2)^2 + (y+7)^2 = 36$ **h)** $(x-2)^2 + (y+5)^2 = 121$

i) $(x-5)^2 + (y+13)^2 = 16$ **j)** $(x+1)^2 + (y-4)^2 = 4$

k) $(x-6)^2 + (y+6)^2 = 64$ **l)** $(x+3)^2 + (y-1)^2 = 25$

Q24

a) $Centre\ (1,3)\ \ radius\ 4$ **b)** $Centre\ (11,-2)\ \ radius\ 6$

c) $Centre\ (2,-4)\ \ radius\ 5$ **d)** $Centre\ (-7,3)\ \ radius\ 8$

e) $Centre\ (-1,3)\ \ radius\ 8\sqrt{2}$ **f)** $Centre\ (6,-1)\ \ radius\ 4\sqrt{6}$

g) $Centre\ (-5,-7)\ \ radius\ 7$ **h)** $Centre\ (3,4)\ \ radius\ \sqrt{17}$

i) $Centre\ (-2,9)\ \ radius\ 9$ **j)** $Centre\ (13,-11)\ \ radius\ 2\sqrt{26}$

k) $Centre\ (1,-6)\ \ radius\ 2\sqrt{6}$ **l)** $Centre\ (8,-1)\ \ radius\ 3\sqrt{6}$

Q25

a) $x^2 + (y-3)^2 = 18$ **b)** $(x+1)^2 + (y-2)^2 = 61$

c) $(x+7)^2 + (y-4)^2 = 90$ **d)** $(x+3)^2 + (y-1)^2 = 122$

e) $(x-3)^2 + (y-2)^2 = 5$ **f)** $(x-2)^2 + (y+9)^2 = 5$

g) $(x-3)^2 + (y-1)^2 = 13$ **h)** $(x-2)^2 + (y+3)^2 = 26$

i) $(x+2)^2 + (y+3)^2 = 45$ **j)** $(x-5)^2 + y^2 = 25$

k) $(x-3)^2 + (y+5)^2 = 29$ **l)** $(x-8)^2 + (y-1)^2 = 185$

Q26 **a)** $Centre\ (2,4)\ \ radius\ 2$ **b)** $Centre\ (4,1)\ \ radius\ 2$

c) $Centre\ (1,-1)\ \ radius\ 3$ **d)** $Centre\ (-2,-3)\ \ radius\ \sqrt{10}$

e) $Centre\ (-1,-1)\ \ radius\ 3$ **f)** $Centre\ (-3,4)\ \ radius\ 1$

g) $Centre\ (-1,1)\ \ radius\ 2$ **h)** $Centre\ (3,1)\ \ radius\ 3$

i) $Centre\ (5,-4)\ \ radius\ 3$ **j)** $Centre\ (4,3)\ \ radius\ 4$

Q27 **a)** YES $r_1 = \sqrt{20}\ \ , r_2 = \sqrt{13}\ \ , Distance\ between\ centres = \sqrt{40}$

$r_1 + r_2 \approx 8.078\ \ , Distance\ between\ centres \approx 6.325$

$8.078 > 6.325\ \ so\ YES\ they\ do\ intersect$

b) YES $r_1 = \sqrt{17}\ \ , r_2 = \sqrt{10}\ \ , Distance\ between\ centres = \sqrt{20}$

$r_1 + r_2 \approx 7.285\ \ , Distance\ between\ centres \approx 4.472$

$7.285 > 4.472\ \ so\ YES\ they\ do\ intersect$

c) NO $r_1 = \sqrt{24}\ \ , r_2 = \sqrt{14}\ \ , Distance\ between\ centres = \sqrt{157}$

$r_1 + r_2 \approx 8.641\ \ , Distance\ between\ centres \approx 12.530$

$8.641 < 12.530\ \ so\ NO\ they\ do\ not\ intersect$

d) YES $r_1 = \sqrt{14}\ \ , r_2 = 6\ \ , Distance\ between\ centres = \sqrt{52}$

$r_1 + r_2 \approx 9.742\ \ , Distance\ between\ centres \approx 7.211$

$9.742 > 7.211\ \ so\ YES\ they\ do\ intersect$

e) YES $r_1 = \sqrt{21}\ \ , r_2 = \sqrt{6}\ \ , Distance\ between\ centres = \sqrt{29}$

$r_1 + r_2 \approx 7.032\ \ , Distance\ between\ centres \approx 5.385$

$7.032 > 5.385\ \ so\ YES\ they\ do\ intersect$

f) NO $r_1 = \sqrt{52}\ \ , r_2 = \sqrt{250}\ \ , Distance\ between\ centres = \sqrt{544}$

$r_1 + r_2 \approx 23.022\ \ , Distance\ between\ centres \approx 23.323$

$23.022 < 23.323\ \ so\ NO\ they\ do\ not\ intersect$

g) YES $r_1 = \sqrt{28}$, $r_2 = \sqrt{50}$, $Distance\ between\ centres = \sqrt{50}$

$r_1 + r_2 \approx 12.363$, $Distance\ between\ centres \approx 7.071$

$12.363 > 7.071$ *so YES they do intersect*

h) NO $r_1 = \sqrt{57}$, $r_2 = \sqrt{20}$, $Distance\ between\ centres = \sqrt{8}$

$r_1 + r_2 \approx 12.022$, $Distance\ between\ centres \approx 2.828$

$r_1 - r_2 \approx 3.078$, *Smaller circle fits within the larger circle so does not touch*

$3.078 > 2.828$ *so NO they do not intersect*

Q28 **a)** $y - 4 = 2(x - 0)$ **b)** $y + 4 = \frac{2}{5}(x - 0)$ **c)** $y - 8 = \frac{4}{11}(x + 6)$

d) $y + 1 = \frac{1}{2}(x + 8)$ **e)** $y + 6 = -\frac{5}{4}(x + 3)$ **f)** $y + 7 = \frac{2}{15}(x - 1)$

g) $y - 5 = -7(x + 3)$ **h)** $y + 7 = -\frac{4}{5}(x + 5)$ **i)** $y + 8 = 7(x - 2)$

j) $y + 0 = -\frac{7}{4}(x - 1)$ **k)** $y + 7 = -\frac{13}{3}(x + 3)$ **l)** $y + 10 = \frac{5}{11}(x - 9)$